Snow College Mathematics Contest

key

April 1, 2014 **Senior Division: Grades 10-12** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Simplify. $4^{\log_2(2^{1/4}\cdot 2^{1/8}\cdot 2^{1/16}\ldots)}$

   (A) 1

   (B) $\sqrt{2}$

   [(C)] 2

   (D) $2\sqrt{2}$

   (E) 4

   SOLN $2^{1/4}\cdot 2^{1/8}\cdot 2^{1/16}\ldots = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots}$

   $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}$

   $4^{\log_2 2^{1/2}} = 4^{1/2} = 2 \quad (\log_b b^x = x)$ □

2. The Pauli spin matrices σ_1, σ_2, and σ_3 appear in quantum mechanics. They are

   $$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

   The inverse of a matrix A is A^{-1} such that $AA^{-1} = A^{-1}A = I$. What is $(\sigma_2)^{-1}$?

   (A) σ_1

   [(B)] σ_2

   (C) σ_3

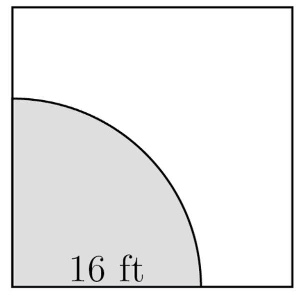
   (D) $-\sigma_2$

   (E) I

   SOLN $\begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}\begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

   Note: $(\sigma_i)^{-1} = \sigma_i$ for $i = 1, 2, 3$; that is, each is its own inverse. □

3. A sprinkler in the corner of a large square lawn sprays a radius of 16 ft. Approximately how many square feet of lawn are watered by the sprinkler?

   (A) $800\,\mathrm{ft}^2$

   (B) $400\,\mathrm{ft}^2$

   (C) $256\,\mathrm{ft}^2$

   [(D)] $200\,\mathrm{ft}^2$

   (E) $64\,\mathrm{ft}^2$

   SOLN $\frac{\pi(16\,\mathrm{ft})^2}{4} = \frac{\pi(256\,\mathrm{ft}^2)}{4} \approx 200\,\mathrm{ft}^2$ □

4. What is the output of the following BASIC computer program?

```
10 dim V(7) :  rem dimension array
20 R = 1
30 V(0) = 1 :  print V(0)
40 V(1) = 2 * R : print V(1)
50 for n = 2 to 7
60 V(n) = (2 * pi * R^2/n) * V(n-2)
70 print V(n)
80 next n
```

   (A) velocity in a collision of n objects

   (B) viscosity of a liquid at temperature n

   (C) nth vibrational mode of an oscillator

   (D) nth vaporization state of a gas

   [(E)] volume of an n-dim unit sphere

   SOLN This is a recursion relation for the volume of an n-dimensional sphere; the function peaks at $n = 5$ and then $V(n) \to 0$ in the limit $n \to \infty$. □

5. One solution of $4x^2 + bx - 3 = 0$ is $\frac{9}{4}$. Find b and the other solution.

(A) $b = -\frac{9}{4}$; $x = \frac{1}{3}$

(B) $b = -9$; $x = -\frac{4}{3}$

(C) $b = -\frac{23}{3}$; $x = -\frac{1}{3}$

(D) $b = \frac{23}{3}$; $x = -\frac{1}{3}$

(E) $b = \frac{9}{4}$; $x = \frac{1}{3}$

SOLN Since one solution is $\frac{9}{4}$ we know that one factor on the left must be $\left(x - \frac{9}{4}\right)$. Since the product of the first terms must be $4x^2$, then the first term of the second factor must be $4x$. Since the product of the last terms in each binomial factor must be -3, the last term of the second factor must be $\frac{4}{3}$. Thus we have $\left(x - \frac{9}{4}\right)\left(4x + \frac{4}{3}\right) = 0$. So $x = -\frac{1}{3}$ to make the second factor zero. Multiply the binomials together: $4x^2 - \frac{23}{3}x - 3 \implies b = -\frac{23}{3}$. □

6. A man born in the first half of the nineteenth century was x years old in the year x^2. In what year was he born?

(A) 1806

(B) 1812

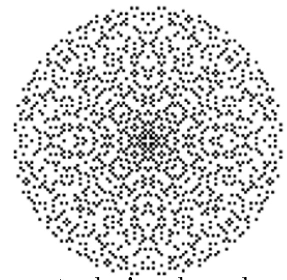
(C) 1825

(D) 1836

(E) 1849

SOLN Look for a whole number x for which $x^2 - x$ is between 1800 and 1850. Note that $40^2 = 1600$ is too small, so try $(40 + \Delta)^2$: $43^2 = 1849$. $1849 - 43 = 1806$ □

7. Gaussian integers are complex $a + b\mathrm{i}$ with $a, b \in \mathbb{Z}$. Some ordinary primes, such as 3, 7, 11, 19, 23, 31, 43, ... are also prime in the Gaussian integers; but 5 is not a Gaussian prime because $5 = (2 + \mathrm{i})(2 - \mathrm{i})$. Which ordinary primes a are also Gaussian primes?



(A) $a = 2^n + 1$

(B) $a = 2n - 1$

(C) $a = b$

(D) $a = 3n + 1$

(E) $a \equiv 3 \pmod 4$

SOLN Find the correct choice by plugging in the list of ordinary primes which are also Guassian primes. See the distribution of Gaussian primes in the complex plane. □

8. Find the sum of the three cube roots of 1.

(A) 0

(B) $\frac{1}{2}$

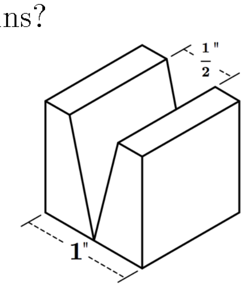
(C) 1

(D) 2

(E) 3

SOLN Cube roots of 1: $\{1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}\}$. They are spread evenly around the unit circle in the complex plane. □

9. A wedge is removed from the center of a cube as shown. How much of the original volume of the cube remains?



(A) $\frac{3}{5}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{\sqrt{2}}{2}$

(E) $\frac{4}{5}$

SOLN $V_{\text{cube}} - V_{\text{wedge}} = 1 - \frac{1}{2}(\frac{1}{2} \cdot 1) = \frac{3}{4}$. Or turn over the piece on the right and join it to the piece on the left to produce a rectangular solid that is 3/4 as wide as the cube was. □

10. How many factors does 1 000 000 have?

(A) 48

(B) 49

(C) 50

(D) 51

(E) 52

SOLN The number of factors of N is $(q_1+1)(q_2+1)\dots(q_n+1)$ where the qs are the multiplicities of the prime factors: $N = p_1^{q_1} p_2^{q_2} \dots p_n^{q_n}$. Since $1\,000\,000 = 2^6 5^6$ then the # of factors is $(6+1)(6+1)$. □

11. Which of the following fractions cannot be written as a terminating decimal?

(A) $\frac{7}{8}$

(B) $\frac{11}{250}$

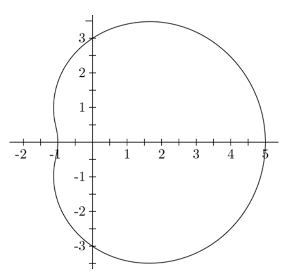
(C) $\frac{11}{125}$

(D) $\frac{37}{768}$

(E) $\frac{99}{256}$

SOLN To be writable as a terminating decimal (base ten) a fraction in lowest terms must have a denominator whose prime factorization contains only factors of 10. $768 = 2^8 \cdot 3$ □

12. Which polar equation best represents the graph for $0 \le \theta \le 2\pi$?



(A) $r = 3 + 2\cos\theta$

(B) $r = 2 + 3\cos\theta$

(C) $r = 2 + 3\sin\theta$

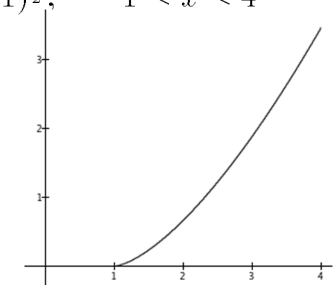
(D) $r = 3 + 2\sin\theta$

(E) $r = 3 + 3\sin\theta$

SOLN Plot points in polar coords. □

13. What is the length of the curve?

$$y = \tfrac{2}{3}(x-1)^{\frac{3}{2}}, \qquad 1 < x < 4$$



(A) $\frac{12\sqrt{3}}{5}$

(B) 9

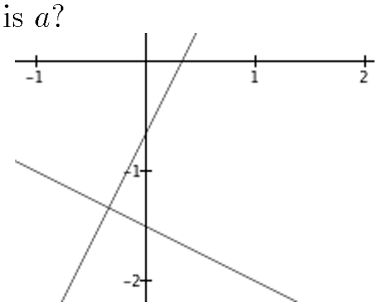
(C) $\frac{14}{3}$

(D) 12

(E) $2\sqrt{3}$

SOLN $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_1^4 \sqrt{1 + (x-1)}\,dx = \left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{2}{3}[8-1] = \frac{14}{3}$ □

14. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, then what is a?



(A) -6

(B) $-\frac{2}{3}$

(C) $-\frac{1}{2}$

(D) $\frac{3}{2}$

(E) 6

SOLN Put each in slope-intercept form: $y = -\frac{1}{2}x - 3$ and $y = -\frac{a}{3}x - 2$. For perpendicular lines the slopes must be negative reciprocals, so $-\frac{a}{3} = 2$. □

15. The empty set is a subset of every set. A set is also a subset of itself. The *power set* of a set S, $\mathcal{P}(S)$, is the set of all subsets of S. How many elements are there in $\mathcal{P}(S)$ if $S = \{$R, O, Y, G, B, I, V$\}$?

(A) 124

(B) 125

(C) 126

(D) 127

(E) 128

SOLN If S has n elements, then $\mathcal{P}(S)$ has 2^n elements. By the way, S is the colors of the rainbow (in order). □

16. The perimeter of a certain right triangle is $12 + 8\sqrt{3}$. The sum of the squares of all three of its sides is 294. Find its area.

(A) $11 + \sqrt{3}$

(B) $\frac{147}{4}$

(C) $10\sqrt{3}$

(D) $6\sqrt{3}$

(E) $24\sqrt{3}$

SOLN $a + b + c = 12 + 8\sqrt{3}$
$a^2 + b^2 + c^2 = 294 \qquad a^2 + b^2 = c^2$
$2c^2 = 294 \Rightarrow c^2 = 147 \Rightarrow c = 7\sqrt{3}$
$a^2 + b^2 = 147$
First equation $\Rightarrow a + b = 12 + \sqrt{3}$
$(a + b)^2 = (12 + \sqrt{3})^2 \Rightarrow 2ab = 24\sqrt{3}$
$A = \frac{ab}{2} = \frac{24\sqrt{3}}{4} = 6\sqrt{3}$ □

17. This problem involves numbers written in two bases we'll call base A and base B. What is the base 10 value of $A + B$ if $31_A = 2_A \cdot 17_A$ and $44_B = 3_B \cdot 13_B$?

(A) 20

(B) 18

(C) 14

(D) 11

(E) 8

SOLN Use positional notation to interpret each equation in base ten.
$3A + 1 = 2(A + 7) \implies A = 13_{\text{ten}}$
$4B + 4 = 3(B + 3) \implies B = 5_{\text{ten}}$
As a check, we knew $A \geq 8$ and $B \geq 5$ (else 17_A and 44_B make no sense). □

18. $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2$ is true for what value of x?

(A) 2

(B) $\sqrt{2}$

(C) $\sqrt[4]{2}$

(D) $2^{\sqrt{2}}$

(E) ∞

SOLN Take ln of both sides.

$$\ln x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = \ln 2$$

The power rule gives

$$x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} \cdot \ln x = \ln 2 \Rightarrow$$

$$x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = \frac{\ln 2}{\ln x} = \log_x 2$$

But $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2$, so

$$2 = \log_x 2 \Rightarrow x^2 = 2$$

□

19. An inlet pipe can fill an empty tank by itself in 2 hours and an outlet pipe can drain the same full tank by itself in 5 hours. If the tank is half full when both valves are opened how long will it take to fill the tank?

(A) 1 h, 50 min

(B) 1 h, 40 min

(C) 1 h, 28 min

(D) 1 h, 18 min

(E) 1 h, 6 min

SOLN Let x be the required time in hours. $\frac{x}{2} - \frac{x}{5} = \frac{1}{2}$. Multiply by a common denominator: $5x - 2x = 5 \Rightarrow$
$x = \frac{5}{3}\,\text{h} = 1\,\text{h}, 40\,\text{min}$. □

20. Where does the line tangent to the sine function at $(\pi, 0)$ intersect the y-axis?

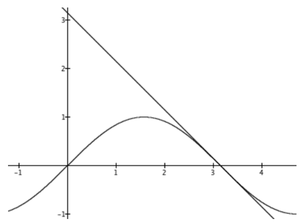
(A) $(0, 0)$

[(B)] $(0, \pi)$

(C) $(0, 1)$

(D) $(0, \frac{\pi}{2})$

(E) $(0, 2\pi)$



[SOLN] The derivative of the sine function is the cosine function. $\cos(\pi) = -1$. Now we have the slope of the tangent line and one point it goes through. $y - 0 = (-1)(x - \pi)$. In slope-intercept form this is $y = -x + \pi$. □

21. At her birthday party Mrs. B was asked her age. She replied that the total of her age and the age of her husband is 140. Then she added "My husband is twice the age I was when he was my age." What is the product of Mr. and Mrs. B's ages?

(A) 4500

(B) 4756

[(C)] 4800

(D) 4875

(E) 4891

[SOLN] Define variables.
w = wife's current age
h = husband's current age
w' = wife's prior age
$h' = w$ = husband's prior age
Same rate of aging: $h - h' = w - w'$
$w' = w - (h - h') = 2w - h$
The problem gives us two equations:
1) $w + h = 140$ and 2) $h = 2w'$
$h = 2w' = 2(2w - h) = 4w - 2h \implies$
$h = \frac{4}{3}w$
$w + h = 140 \implies w + \frac{4}{3}w = 140 \implies$
$w = 60 \qquad h = \frac{4}{3}w = \frac{4}{3}60 = 80$ □

22. A triangle has sides of lengths 8.1 and 1.4. What is the length of the third side, if it is an even integer?

(A) 2

(B) 4

(C) 6

[(D)] 8

(E) 10

[SOLN] Triangle inequality: $8.1 - 1.4 < c < 8.1 + 1.4 \implies 6.7 < c < 9.5$ □

23. Find $b - a$ if (a, b) is the solution to the system of equations.

$$\begin{cases} \pi a + (\pi + \mathrm{e})b = \pi + 2\mathrm{e} \\ (\pi + 3\mathrm{e})a + (\pi + 4\mathrm{e})b = \pi + 5\mathrm{e} \end{cases}$$

(A) -3

(B) -1

(C) 0

(D) 1

[(E)] 3

[SOLN] $R_2 - R_1 \implies a + b = 1$ (The π part of either equation gives same.) The e part of the first equation gives $b = 2$. □

24. Simplify the expression: $\dfrac{\tan t - \sin t \cos t}{\tan t}$

(A) $\sin t$

(B) $\cos t$

[(C)] $\sin^2 t$

(D) $\cos^2 t$

(E) 1

[SOLN] Turn the $\tan t$ into $\sin t / \cos t$ and simplify. □

25. Randomly choose k digits (with repetitions allowed) from $\{0,1,2,3,4,5,6,7,8,9\}$. What is the probability that 0 will *not* be chosen?

(A) $\frac{1}{k}$
(B) $\frac{1}{10}$
(C) $\frac{k}{k-1}$
(D) $\left(\frac{1}{10}\right)^k$
[(E)] $\left(\frac{9}{10}\right)^k$

SOLN The probability that any one of the digits is not 0 is $\frac{9}{10}$. □

26. How many minutes after noon is the first time that the hour and minute hands are pointing in exactly opposite directions?

(A) 32
(B) $32\frac{1}{2}$
(C) $32\frac{7}{12}$
[(D)] $32\frac{8}{11}$
(E) 33

SOLN Let $h(t) = \frac{2\pi}{12}t$ and $m(t) = 2\pi t$ be the number of radians the hour and minute hands pass through in t hours since noon. The answer in hours is the solution of $h(t) + \pi = m(t)$. Solve this to get $t = \frac{6}{11}$ h $= \frac{360}{11}$ min. □

27. In one lottery there is a bubble sheet of numbers from 01–60 and a ticket is completed by filling in any five of those numbers. If you randomly pick the five numbers, what are the chances you will win the jackpot?

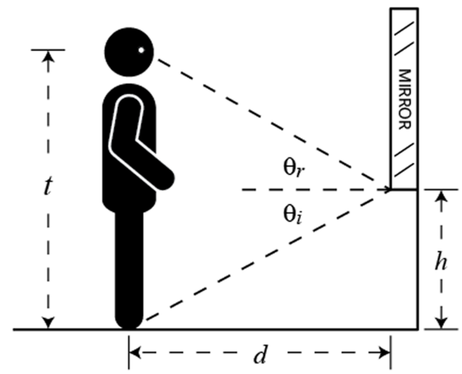
(A) $\frac{5!}{60!}$
(B) $\frac{5!}{55!}$
[(C)] $\frac{5! \cdot 55!}{60!}$
(D) $\frac{1}{5!}$
(E) $\frac{50!}{55!}$

SOLN Since order doesn't matter, this is a combination of 60 items taken 5 at a time. □

28. Let the bottom edge of a rectangular mirror on a vertical wall be parallel to and h feet above the level floor. If a person with eyes t feet above the floor is standing erect at a distance d feet from the mirror, what is the relationship among h, d, and t if the person can just see his own feet in the mirror?

[(A)] $t = 2h$ and d doesn't matter
(B) $t = 4d$ and h doesn't matter
(C) $h^2 + d^2 = \frac{t^2}{4}$
(D) $t - h = d$
(E) $(t-h)^2 = 4d$

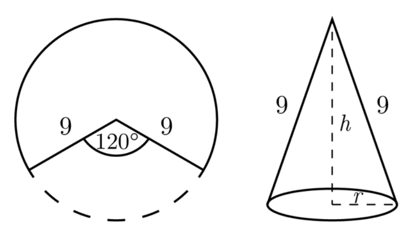
SOLN



Draw a diagram. Angle of reflection equals angle of incidence. □

29. A piece of paper has the shape of the larger circular sector with dimensions and angles as shown. This paper is suitably folded to form the vertical cone shown at right. Find the height h of the cone.

(A) 3
[(B)] $3\sqrt{5}$
(C) $3\sqrt{2}$
(D) $9\sqrt{2}$
(E) 9



SOLN The circumference of the circular sector is the circumference of the base of the cone. Solve for the radius of the cone. $2\pi r = \frac{240°}{360°} \times 2\pi \times 9 \implies r = 6$ Use the Pythagorean theorem on the triangle h-r-9. $h = \sqrt{9^2 - 6^2}$ □

30. It's Sophie's birthday! Sophie Germain, famous, self-taught woman mathematician was born April 1, 1776. The identity named after her says which of the following is equivalent to $x^4 + 4y^4$?

(A) $(x+y)(x+y)(x-y)(x-y)$

(B) $(x^2+2xy+2y^2)(x^2-2xy+2y^2)$

(C) $(x+y)^2(x+2y)^2$

(D) $(x+y)^2(x-y)(x+4y)$

(E) $(x-y)^2(x+y)(x+4y)$

SOLN Multiply the two trinomials and collect like terms.
$((x+y)^2+y^2)((x-y)^2+y^2)$ □

31. What is the real number equivalent of i^{2i}?

(A) e^{-1}

(B) $e^{-\frac{\pi}{2}}$

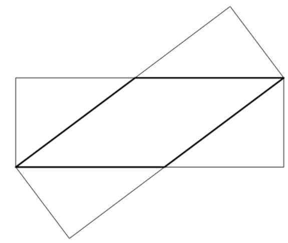
(C) $e^{-\pi}$

(D) $e^{-2\pi}$

(E) No real number equivalent exists

SOLN $i^{2i} = e^{2i\ln i} = e^{2i\left(\ln e^{\frac{i\pi}{2}}\right)} = e^{2i\left(\frac{i\pi}{2}\right)}$
Or $i^{2i} = \left(e^{i\frac{\pi}{2}}\right)^{2i} = e^{i^2\pi} = e^{-\pi}$ □

32. A copy of a $6\,\text{cm} \times 18\,\text{cm}$ rectangle is placed on top of the original rectangle as shown. What is the area of the parallelogram formed by the intersection?



(A) $54\,\text{cm}^2$

(B) $12\sqrt{34}\,\text{cm}^2$

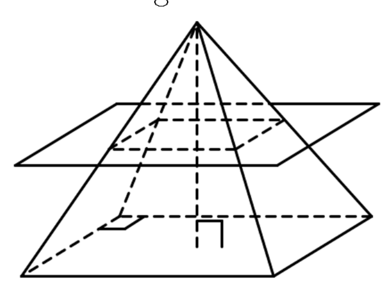
(C) $18\sqrt{10}\,\text{cm}^2$

(D) $36\sqrt{2}\,\text{cm}^2$

(E) $60\,\text{cm}^2$

SOLN There are four congruent right triangles. Use Pythagorean theorem: $6^2 + (18-x)^2 = x^2 \implies x = 10$
Therefore the parallelogram has a base of 10 cm and a height of 6 cm.
Or: the area of the parallelogram will be half of (sum of the two rectangles minus the four right triangles) = $\frac{1}{2}[2\cdot 6\cdot 18 - 4\cdot\frac{1}{2}(6\cdot 8)] = \frac{1}{2}[216-96]$ □

33. A right square pyramid is cut by a plane parallel to its base halfway up the altitude, creating a smaller version of the pyramid. What is the ratio of the volume of the small pyramid to that of the original?



(A) $1:2$

(B) $1:4$

(C) $1:6$

(D) $1:8$

(E) $1:10$

SOLN Since the smaller pyramid is similar to the original, it is half as tall, half as wide, and half as deep. The volume of the smaller is $\left(\frac{1}{2}\right)^3 \cdot V_{\text{orig}}$. □

34. Find the equation of the line through the origin that bisects the area of the unit square shown.

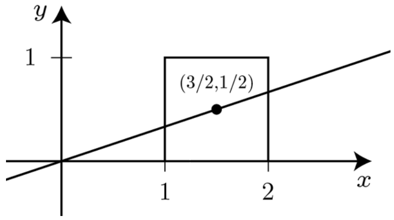
(A) $y = \frac{1}{3}x$

(B) $y = \frac{1}{2}x$

(C) $y = \frac{2}{3}x$

(D) $y = \frac{3}{2}x$

(E) $y = 2x$



SOLN To bisect a square a straight line must pass through its center. The center of this square is $(\frac{3}{2}, \frac{1}{2})$. Since the line goes through $(0,0)$ the intercept is 0 and $m = \frac{1/2}{3/2}$. □

35. Let f be a function satisfying

$$f(xy) = \frac{f(x)}{y}$$

If $f(500) = 3$, what is $f(600)$?

(A) $\frac{2}{5}$

(B) $\frac{5}{6}$

(C) $\frac{6}{5}$

(D) $\frac{5}{2}$

(E) $\frac{18}{5}$

SOLN Let $x = 500$ and $y = \frac{6}{5}$.
$f(600) = f(500 \cdot \frac{6}{5}) = \frac{f(500)}{6/5} = \frac{3}{6/5}$
Can you see why $f(x) = \frac{1500}{x}$? □

36. Positive integers a, b, c, with no common factor greater than 1, exist such that $a \log_{200} 5 + b \log_{200} 2 = c$. What is the sum $a + b + c$?

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

SOLN $a \log_{200} 5 + b \log_{200} 2 =$
$\log_{200} 5^a + \log_{200} 2^b =$
$\log_{200}(5^a \cdot 2^b) = c \implies 5^a \cdot 2^b = 200^c$
$\implies a = 2,\ b = 3,\ c = 1$ □

37. If $\sin x = 3 \cos x$, what is $\sin x \cos x$?

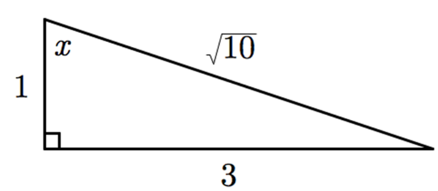
(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{2}{9}$

(D) $\frac{1}{4}$

(E) $\frac{3}{10}$



SOLN Note: since $\sin x$ and $\cos x$ have the same sign, the angle lies in the 1st or 3rd quadrant, so $\sin x \cos x > 0$.
$\sin x = 3 \cos x \implies \tan x = 3$.
From the triangle we see that
$\sin x = \frac{3}{\sqrt{10}}$ and $\cos x = \frac{1}{\sqrt{10}}$. □

38. Five girls wish to give each other gifts. Each girl puts her name in a hat and then each draws a name from the hat. In how many ways can the names be drawn from the hat so that no one draws her own name?

(A) 24

(B) 25

(C) 44

(D) 86

(E) 120

SOLN This question is asking for all the *derangements* (permutations of an ordered list of symbols in which no symbol is in its natural position) of 5 symbols. One may count the number of derangements directly (long) or deduce the following formula: $D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right)$ □

39. Which is true if the lines with equations $y = 2x + b$ and $y = mx - 6$ intersect at a point on the x-axis?

(A) $mb = 12$

(B) $mb + 12 = 0$

(C) $m = 3b$

(D) $m + 3b = 0$

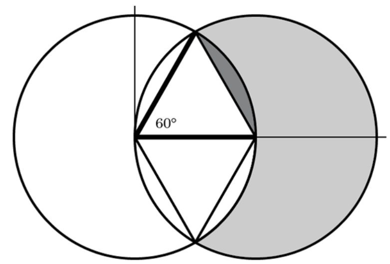
(E) $3m = b$

SOLN Since the two lines intersect at a point $(a, 0)$, solve the system

$$\begin{cases} 2a + b = 0 \\ ma - 6 = 0 \end{cases}$$

□

40. What is the area shared by two intersecting circles of radius 1 passing through each other's center?



(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

(C) $\frac{\sqrt{3}}{2} + \frac{\pi}{18}$

(D) $\frac{\pi}{4}$

(E) $\frac{\pi}{3}$

SOLN Geometry: Consider one wedge from the center of the circle on the left, the inscribed equilateral triangle, and the left-over lunate sliver. $A_{\text{wedge}} = \frac{\pi}{6}$ and $A_{\text{triangle}} = \frac{\sqrt{3}}{4}$, so $A_{\text{sliver}} = \frac{\pi}{6} - \frac{\sqrt{3}}{4}$. The total area is two equilateral triangles plus four slivers: $A_{\text{tot}} = 2 \cdot \frac{\sqrt{3}}{4} + 4 \cdot \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$.

Calculus: Divide the total area into four equal parts by lines of horizontal and vertical symmetry.

$$A_{\text{tot}} = 4\int_{1/2}^{1} \sqrt{1 - x^2}\, dx$$

$$= 4\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1} x\right]_{1/2}^{1}$$

$$= 4\left[0 + \frac{1}{2}\frac{\pi}{2} - \frac{\frac{1}{2}\sqrt{\frac{3}{4}}}{2} - \frac{1}{2}\frac{\pi}{6}\right]$$

Calculus: Express the circles in polar coords: $r = 1$ and $r = 2\cos\theta$. They intersect where $\cos\theta = \frac{1}{2}$. The area of overlap is the area of the right circle minus the area not in the left circle.

$$A = \pi(1)^2 - \int_{-\pi/3}^{\pi/3} \tfrac{1}{2}\left[(2\cos\theta)^2 - 1^2\right]\, d\theta$$

$$= \pi - \int_{-\pi/3}^{\pi/3} \left(2\cos^2\theta - \tfrac{1}{2}\right)\, d\theta$$

$$= \pi - \int_{-\pi/3}^{\pi/3} \left(\cos 2\theta + 1 - \tfrac{1}{2}\right)\, d\theta$$

$$= \pi - \left[\frac{\sin 2\theta}{2} + \frac{1}{2}\theta\right]_{-\pi/3}^{\pi/3}$$

□