Snow College Mathematics Contest   April 4, 2006   Form: $\boxed{\text{A}}$

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.
2. Go ahead now and fill in the box at the top of your answer sheet. Print your name clearly, put your phone number in the "ID#" blank, spell out your school in the "class" blank, and put your year in school in the "sec" blank. Put your test version (Form A) in the "test no." blank. Also use a #2 (or HB or soft) pencil to bubble in your name on the left side of the answer sheet.
3. This is a two hour examination consisting of 40 multiple choice problems. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is Score $= 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from -40 to 160.
4. In the event of a tie, the person with the largest number of the following five problems correct will be delcared the winner: 2, 13, 27, 30, 37. Any further ties will be broken by a coin toss.
5. When the test begins, bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Use #2 (or soft) pencil. Erase any incorrect answers completely.
6. The sketches that accompany the problems are not necessarily drawn to scale.
7. No calculators are allowed.
8. Do not talk or disrupt other test takers during the exam. Cell phones must be OFF.
9. Please raise your hand if you need scratch paper; a proctor will assist you.
10. The proctors have been advised to answer no questions pertaining to the exam.
11. While we recommend you stay and recheck your answers if you have time, you may leave if you finish early (if you do, turn your answer sheet in and leave quietly). After the two hour time limit is up the proctors will call for your answer sheets. Hand them in promptly.

After the test:

1. You may keep this test booklet.
2. If you RSVP'd to spend time with one of our science departments for lunch, please meet them in the science building; otherwise lunch may be purchased at the Snow College Cafeteria or downtown. In any event, you should plan to be back at the LDS Institute by 1:30 p.m. for the scores and presentation of the awards.
3. The top three scorers from each classification of school will receive full tuition scholarships to Snow College. Other prizes will be awarded to other individuals.
4. Thanks for coming. Your instructors will be happy to work the problems for you, and they will also be given your corrected answer sheets.

Snow College Mathematics Contest April 4, 2006 Form: $\boxed{\text{A}}$ $\boxed{\text{key}}$

1. When $x = 10$, the expression $\sqrt{1+2+3+x}$ has the value 4. What is the sum of all four integers $x < 10$ for which $\sqrt{1+2+3+x}$ has an integer value?

   $\boxed{\text{(A)}}$ -10

   (B) -6

   (C) 6

   (D) 10

   (E) -4

   By definition $\sqrt{1+2+3+x} \geq 0$. Since $x < 10$, it follows that $0 \leq \sqrt{6+x} < 4$. The answers are the solutions to $\sqrt{6+x} = 0$, $\sqrt{6+x} = 1$, $\sqrt{6+x} = 2$, $\sqrt{6+x} = 3$. These asnwers are $-6, -5, -2, 3$.

2. A box contains 4 fair coins and 6 biased coins. Whenever a fair coin is flipped, it comes up heads with a probability 0.5. Whenever a biased coin is flipped, it comes up heads with probability 0.8. A coin is randomly chosen from the box and then flipped. What is the probability that it will come up heads?

   (A) 0.6

   (B) 0.64

   $\boxed{\text{(C)}}$ 0.68

   (D) 0.72

   (E) 0.76

   The probability that the random coin is a fiar coin and that it comes up heads is $0.4 \cdot 0.5 = 0.2$. The probability that the random coin is a biased coin and that it comes up heads is $0.6 \cdot 0.8 = 0.48$. Thus, the probability that the random coin comes up heads is $0.2 + 0.48 = 0.68$.

3. What is the value of the product $(\log_2 3)(\log_3 5)(\log_5 8)$?

   (A) 2

   $\boxed{\text{(B)}}$ 3

   (C) 4

   (D) 5

   (E) 6

   $(\log_2 3)(\log_3 5)(\log_5 8) = (\log_2 3) \cdot \frac{\log_2 5}{\log_2 3} \cdot \frac{\log_2 8}{\log_2 5} = \log_2 8 = 3$

4. Both x and y are positive real numbers less than 2. Every positive number less than 2 is equally likely to be the value of x; and every positive number less than 2 is equally likely to be the value of y. What is the probability that x and y differ by less than 1?

   (A) 0.20

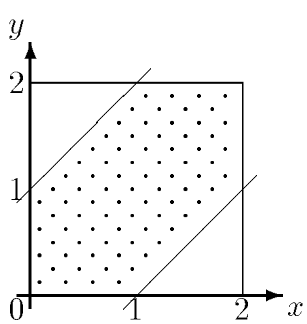
   (B) 0.25

   (C) 0.50

   (D) 0.65

   $\boxed{\text{(E)}}$ 0.75

   Since $0 < x < 2$ and $0 < y < 2$, we can use the coordinate plane to model the given conditions. The graph of the model is a 2×2 square in the first quadrant. Every point (x, y) inside the square meets the requirements that x and y are both positive and less than 2. The values of x and y differ by less than 1 everywhere where $|x = y| < 1$, that is, only if $y > x - 1$ and $y < x + 1$. The unshaded region f the 2×2 square can be reassembled to form a 2×2 square, so its area is 1. The required probability is the fractional part of the square that is shaded. This probability, which is the area of the shaded region divided by the area of the square, equals 0.75.



5. If the lengths of two sides of a right triangle are 3 and 4, what is the least possible length of the third side?

   $\boxed{\text{(A)}}$ $\sqrt{7}$

   (B) 3

   (C) 4

   (D) 5

   (E) 7

   There are two possibilities: the lengths of the legs could be 3 and 4 (making the length of the hypoteneuse 5), or the length of one leg could be 3 and the length of the hypoteneuse could be 4, making the length of the third side $\sqrt{4^2 - 3^2} = \sqrt{7}$.

6. The sides of a triangle are in the ratio $3 : 5 : 9$. Which of the following words best describes the triangle?

   (A) obtuse

   (B) scalene

   (C) right

   (D) isosceles

   $\boxed{\text{(E)}}$ impossible

   The triangle inequality, which states that the sum of any two sides of a triangle is at least as large as the third side, can be invoked. There are no such triangles.

7. Let $f(x) = \sqrt{(x-2)^2}$. Compute $\displaystyle\sum_{x=-2}^{x=2} f(2x)$.

(A) -7

(B) 0

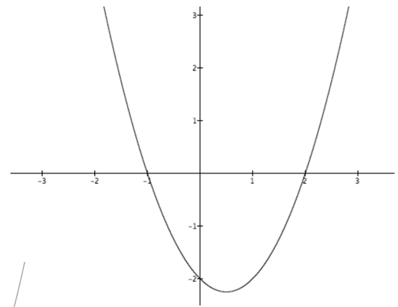
(C) 7

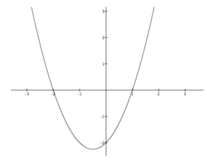
$\boxed{\text{(D)}}$ 14

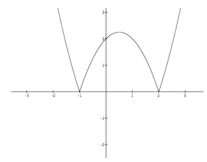
(E) 16

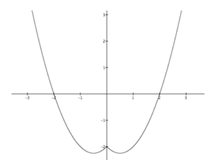
Another way to write f is $f(x) = |x - 2|$, so $f(2x) = |2x - 2|$, and the sum in question is $|2(-2)-2| + |2(-1)-2| + |2(0)-2| + |2(1)-2| + |2(2)-2| = 6+4+2+0+2 = 14$.

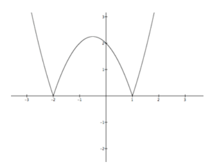
8. Let $f(x)$ be the function whose graph is shown. Which of the following represents the graph of $f(|x|)$?

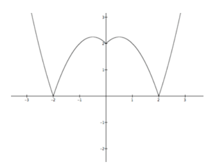


(A) 

(B) 

$\boxed{\text{(C)}}$ 

(D) 

(E) 

The functions shown are A: $f(-x)$, B: $|f(x)|$, C: $f(|x|)$, D: $|f(-x)|$, E: $|f(|x|)|$.

9. If the operation $\oplus$ is defined for all positive x and y by $x \oplus y = (xy)/(x+y)$, which of the following must be true for positive x, y, and z?

   (i) $x \oplus x = x/2$

   (ii) $x \oplus y = y \oplus x$

   (iii) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

   (A) (i) only
   (B) (i) and (ii) only
   (C) (i) and (iii) only
   (D) (ii) and (iii) only
   $\boxed{\text{(E)}}$ all three

   The operation can be written $x \oplus y = (\frac{1}{x} + \frac{1}{y})^{-1}$ which demonstrates (i). It is also clear that it is commutative and associative.

10. On a musical instrument with 4 valves, how many different fingerings are possible?

   (A) 8
   (B) 10
   (C) 12
   $\boxed{\text{(D)}}$ 16
   (E) 20

   Each valve can be up or down (two positions), so the answer is $2^4 = 16$.

11. An explorer wishes to cross a barren desert that requires 6 days to cross, but one man can only carry enough food for 4 days. What is the fewest number of other men required to help carry enough food for him to cross (and everyone stay alive)?

   (A) 1
   $\boxed{\text{(B)}}$ 2
   (C) 3
   (D) 4
   (E) it can't be done with any number of extra men

   First realize that if each of the men ate his own food then no number of extra men will help solve the problem. The idea is to get only the one explorer across and have the helpers turn back. Two are required. The first only goes one day into the desert and feeds all three men the first day and turns back with one day's rations for the return trip. On the second day the second helper feeds himself and the explorer. On the beginning of the third day the helper how has two days rations left and heads back. The explorer is two days into the journey and still has all four days of his food left, so he continues alone.

12. In this problem all numbers are written in base 3. What is 21×21?

$\boxed{\text{(A)}}$ 1211

(B) 1001

(C) 11111

(D) 1121

(E) 441

$21_3 = 7_{10}$ and $7_{10} \times 7_{10} = 49_{10}$. Converting 49_{10} back to base 3 gives $(1 \times 27)+(2\times 9)+(1\times 3)+(1\times 1) = 1211_3$.

Or, staying in base 3:

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 21 \\ +112 \\ \hline 1211 \end{array}$$

13. A square has four corners (or vertices), four edges, and one face. A cube has eight corners, twelve edges, six faces, and one volume. How many edges would a 4-D hypercube (*a.k.a.*, a tesseract) have?

(A) 24

(B) 28

(C) 30

$\boxed{\text{(D)}}$ 32

(E) 36

The number of edges in dimension $n+1$ is $2e+v$ where e and v are the number of edges and vertices (corners) in dimension n. (Copying the figure gives $2e$ and then v more to connect the first copy's corners to the second's.)

14. Where do the the asymptote of $\frac{x^2+5x+6}{x-2}$ and the line with equation $y = 2x+3$ cross?

$\boxed{\text{(A)}}$ Quadrant I

(B) Quadrant II

(C) Quadrant III

(D) Quadrant IV

(E) On an axis

The asymptote is $x = 2$ which is in quadrants I and IV; the line goes through quadrants I, II, and III.

15. What is $\cot(\frac{\pi}{6})$? (Hint: bisect one angle of an equilateral triangle whose sides are length 2.)

(A) 2

(B) $\frac{\sqrt{3}}{3}$

(C) $\frac{2\sqrt{3}}{3}$

(D) $\frac{\sqrt{3}}{2}$

$\boxed{\text{(E)}}$ $\sqrt{3}$

A 30-60-90 triangle has sides 1, $\sqrt{3}$, 2. $\frac{\pi}{6}$ radians $=30°$, and the adjacent over the opposite is $\sqrt{3}/1$. To solve this problem a student must know that the sum of the interior angles of a triangle is 180°, the Pythagorean theorem, how to convert radians to degrees, that the cot is $1/\tan$, and SOH CAH TOA.

16. The n^{th} triangular number is defined as the sum of the first n whole numbers. (One can arrange dots in a triangular pattern with one dot in the first row, two in the second, etc.) Thus, the 4^{th} triangular number is 10 becuase $10 = 1 + 2 + 3 + 4$. Carl Freidrich Gauss' elementary school class was given the task of adding up the the integers from 1 to 100 in the hopes that it would take the students a lot of time. The teacher was astonished when Gauss immediately came up with what value for the 100^{th} triangular number?

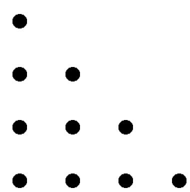
(A) 4900

(B) 4949

(C) 4950

(D) 5000

$\boxed{\text{(E)}}$ 5050



Each number can be paired with another to give a subtotal of 101. For example $1+100 = 101$ and $2+99 = 101$, etc. There are 50 such pairs, so the grand total is $50 \times 101 = 5050$.

17. The left side of the equation is the determinant of the matrix product. The equation

$$\left| \begin{bmatrix} x & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} -2 & x \\ 0 & 0 \end{bmatrix} \right| = 0$$

(A) is satisfied for only one value of x.

(B) is satisfied for two values of x.

(C) is satisfied for no values of x.

$\boxed{\text{(D)}}$ is satisfied for an infinite number of values of x.

(E) None of these.

Multiplying the matrices gives

$$\begin{vmatrix} -2x & x^2 \\ -6 & 3x \end{vmatrix} = -6x^2 + 6x^2 = 0$$

for all x.

18. What is the equation of the line passing through the point $(1, 7)$ and perpendicular to the line with x-intercept 6 and y-intercept 2?

(A) $y = 3x + 6$

(B) $3x + y = 6$

$\boxed{\text{(C)}}$ $-3x + y = 4$

(D) $y = -3x + 4$

(E) $x + 3y = 6$

The line with x-intercept 6 and y-intercept 2 has equation $3y + x = 2$ or $y = -\frac{1}{3}x + 2$. The line we desire must have a slope that is the negative reciprocal of $-\frac{1}{3}$ which is 3, so it's equation is $y = 3x + b$ where b is determined by $f(1) = 7$.

19. If $f(z) = \frac{z^3+2z}{z+1}$ and $\mathrm{i} = \sqrt{-1}$ then what is $f(\mathrm{i})$?

(A) $\mathrm{i} - 1$

(B) $\mathrm{i} + 1$

(C) $1 - \mathrm{i}$

(D) $\sqrt{2}$

$\boxed{\text{(E)}}$ none of the above

$f(\mathrm{i}) = \frac{\mathrm{i}^3+2\mathrm{i}}{\mathrm{i}+1} = \frac{-\mathrm{i}+2\mathrm{i}}{\mathrm{i}+1} = \frac{\mathrm{i}}{(\mathrm{i}+1)}\frac{(\mathrm{i}-1)}{(\mathrm{i}-1)} = \frac{-1-\mathrm{i}}{-1-1} = \frac{-(\mathrm{i}+1)}{-2} = \frac{\mathrm{i}+1}{2}$

20. Two sets are said to have the same cardinality if the elements of one can be put into one-to-one correspondence with the elements of the other. Which of the sets below does not have the same cardinality as at least one of the others? (Hint: think of rules that show the one-to-one correspondences.) (Infinite sets are defined to be ones that can be put into one-to-one correspondence with proper subsets of themselves.)

(A) $\mathbb{N}$, the natural numbers $= \{1, 2, 3, \ldots\}$

(B) $\mathbb{W}$, the whole numbers $= \mathbb{N} + \{0\}$

(C) $\mathbb{Z}$, the integers

(D) $\mathbb{Q}$, the rational numbers

$\boxed{\text{(E)}}$ $\mathbb{R}$, the real numbers

All of the sets have cardinality $\aleph_0$ with the exception of $\mathbb{R}$.

21. Given the sequence $\sqrt{3}, \sqrt{6}, 3, 2\sqrt{3}, \ldots$, which term would be $3\sqrt{2}$?

$\boxed{\text{(A)}}$ 6^{th}

(B) 7^{th}

(C) 10^{th}

(D) 11^{th}

(E) 12^{th}

The pattern is square roots of multiples of three. The sixth term would be $\sqrt{6 \cdot 3} = \sqrt{18} = 3\sqrt{2}$.

22. Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, what is $(g \circ f)(-2)$ where $(g \circ f)$ is the composition of the functions?

(A) -4

(B) 0

(C) 2

$\boxed{\text{(D)}}$ 4

(E) 8

$(g \circ f)(x) = g(f(x)) = g(x + 2) = 4 - (x + 2)^2 = 4 - (x^2 + 4x + 4) = -x^2 - 4x$

23. Long-distance radio navigation for aircraft and ships uses synchronized pulses (which travel at the speed of light) transmitted by two widely separated transmitting stations. The ship or plane uses the differences in arrival times of these pulses to determine its location. Which kind of conic section would best describe the path of such a ship or plane if the differences in pulse arrival times doesn't change over time?

(A) circle

(B) ellipse

(C) parabola

$\boxed{\text{(D)}}$ hyperbola

(E) none of the above

The hyperbola is the locus of points which have a common difference from two foci.

24. Given $f(x) = 2x^3 - 3x^2 - 12x + 18$, at what x value does $f(x)$ have a local minimum?

(A) -1

$\boxed{\text{(B)}}$ 2

(C) 3

(D) 6

(E) none of the above

We take the derivative of $f(x)$ and set it equal to zero. $f'(x) = 6x^2 - 6x - 12 = 0$. Dividing both sides by 6 and factoring gives $(x - 2)(x + 1) = 0$, so the local extrema occur at -1 and 2. To find which one is the local minimum we take the second derivative $f''(x) = 12x - 6$ and check to find that $f''(-1)$ is negative while $f''(2)$ is positive, so $x = 2$ is the abscissa of the local minimum.

25. The binomial coefficient (read "n choose r") is defined by

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

What is $\binom{12}{2}$?

(A) 60

$\boxed{\text{(B)}}$ 66

(C) 120

(D) 47 900 160

(E) 239 500 800

$$\binom{12}{2} = \frac{12!}{2!\,10!} = \frac{12\cdot 11}{2} = 66$$

26. What is the output of the following computer program?

```
10 for i = 1 to 3
20 for j = 1 to i
30 print i*j
40 next j
50 next i
```

$\boxed{\text{(A)}}$ 1, 2, 4, 3, 6, 9

(B) 1, 2, 3, 1, 2, 3

(C) 1, 2, 2, 3, 3, 3

(D) 1, 2, 3, 4, 5, 6, 7, 8, 9

(E) 1, 2, 3, 1, 2, 3, 1, 2, 3

i	j	$i*j$
1	1	1
2	1	2
2	2	4
3	1	3
3	2	6
3	3	9

27. What curve is represented by the parametric equations?

$$\left.\begin{aligned} x &= 2\sin t \\ y &= 3\cos t \end{aligned}\right\} \text{for } t \text{ in } [0, 2\pi]$$

(A) circle

(B) hyperbola, horizontal transverse axis

(C) hyperbola, vertical transverse axis

(D) ellipse, horizontal major axis

$\boxed{\text{(E)}}$ ellipse, vertical major axis

Square each parametric equation, divide each by the numerical coefficient and then use the trig identity $\sin^2 t + \cos^2 t = 1$ to arrive at $\frac{x^2}{4} + \frac{y^2}{9} = 1$ which is an ellipse with a vertical major axis.

28. Two cans (right cylinders) have the same volume. The height of one can is triple the height of the other. If the radius of the narrower can is 12 cm, what is the radius of the wider can?

(A) $2\sqrt{12}$ cm

$\boxed{\text{(B)}}$ $12\sqrt{3}$ cm

(C) $6\sqrt{3}$ cm

(D) $3\sqrt[3]{12}$ cm

(E) $3\sqrt{12}$ cm

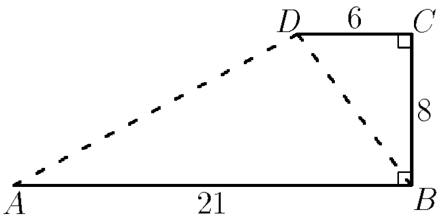
$r_1 = 12 \quad h_1 = 3h_2$

$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$r_2^2 = \frac{r_1^2 h_1}{h_2} = \frac{r_1^2(3h_2)}{h_2}$$

$$r_2 = \sqrt{3r_1^2}$$

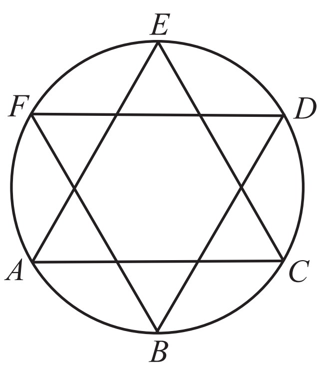
29. What is the sum of the distances AD and BD in the figure?



$\boxed{\text{(A)}}$ 27
(B) 28
(C) 29
(D) 30
(E) 31

By the Pythagorean theorem, $BD = 10$. Let E be the point on $\overline{AB}$ with $\angle DEA = 90°$. Then $DE = 8$ and $AE = 21 - 6 = 15$. By the Pythagorean theorem again, $AD = 17$. Hence the answer is $10 + 17 = 27$.

30. The six points A, B, C, D, E, F are equally spaced along the circumference of a circle of radius 1. Line segments $\overline{AC}, \overline{CE}, \overline{AE}, \overline{BD}, \overline{DF}$, and $\overline{BF}$ are drawn. What is the area of the six-pointed star-shaped region?



(A) $\frac{2\sqrt{6}}{3}$
$\boxed{\text{(B)}}$ $\sqrt{3}$
(C) $\frac{2\pi}{3}$
(D) $\frac{3\sqrt{2}}{2}$
(E) $\frac{3\pi}{4}$

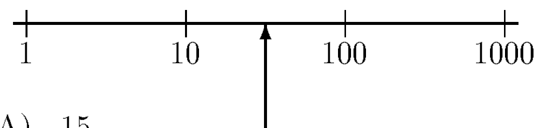
Let O denote the center of the circle, G the intersection of $\overline{EA}$ and $\overline{FD}$, and H the intersection of $\overline{EC}$ and $\overline{FD}$. Since $OE = 1$, the height of the equilateral triangle $\triangle EGH$ is $1/2$. It easily follows that $\triangle EGH$ has side-length $\sqrt{3}/3$ and area $\sqrt{3}/12$. The star-shaped region can be divided up into 12 such equilateral triangles, each congruent to $\triangle EGH$. It follows that the area of the star-shaped region is $\sqrt{3}$.

31. A drawer has 6 red socks and 6 white socks. If you reach in the drawer and randomly take out two socks, what is the chance (*i.e.*, probabilitiy) that the two socks will match in color?

(A) $\frac{5}{12}$
(B) $\frac{1}{2}$
(C) $\frac{2}{5}$
(D) $\frac{3}{7}$
$\boxed{\text{(E)}}$ $\frac{5}{11}$

Say the first sock is red. Then there are 5 remaining red socks and 11 total socks in the drawer, so the chance of matching the first red sock with a second is $\frac{5}{11}$.

32. The diagram shows the scale on a measuring device. What is the approximate reading at the arrow?



(A) 15
$\boxed{\text{(B)}}$ 32
(C) 50
(D) 55
(E) 75

Since we have a logarithmic scale we are looking for a number whose $\log_{10}$ is about 1.5, and that number is 32.

33. What is the sum of the exponents of the prime factorization of 200?

(A) 2
(B) 4
$\boxed{\text{(C)}}$ 5
(D) 7
(E) 8

The prime factorization is $2^3 \cdot 5^2$.

34. By using the answers to the following questions, Chris determines Terry's secret number. What is Terry's secret number?

- Is it a factor of 30? Yes
- Is it a prime number? No
- Is it a multiple of 3? No
- Is it less than 3? No

(A) 3

(B) 5

(C) 6

$\boxed{\text{(D)}}$ 10

(E) 15

The mystery number is a factor of 30, so it can be built up by using the factors 1, 2, 3, or 5. It is not a prime number, so it is not any of the numbers 2, 3, or 5, but it is a product of some or all of these numbers. It is not a multiple of 3, so it does not have a factor of 3. The remaing factors are 1, 2, and 5. It is not less than 3, so it uses the factors 2 and 5. It must therefore be the number 10.

35. If you start with 128 g of carbon-14 (^{14}C), whose half-life is 5700 years, how much ^{14}C will be left after 28 500 years?

(A) 128 g

(B) 64 g

(C) 32 g

$\boxed{\text{(D)}}$ 4 g

(E) Approximately zero

After five half-lives the amount will be decreased by a factor of $2^5 = 32$.

36. What quadrilateral can be divided into three equilateral triangles?

(A) square

(B) rectangle

(C) rhombus

(D) parallelogram

$\boxed{\text{(E)}}$ isosceles trapezoid

One base of the trapezoid is one side of a triangle, and the other base of the trapezoid is twice as long and is sides of the other two triangles.

37. Given $\omega^3 = 1$ and $\omega \neq 1$, what is $\omega(\omega+1)$?

(A) i

(B) $-$i

(C) 1

$\boxed{\text{(D)}}$ -1

(E) $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}$

$\omega = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}; \quad \omega + 1 = \frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$

$(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i})(\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}) = -\frac{1}{4} - \frac{3}{4}$

38. Which function of x and y is represented by the graph?

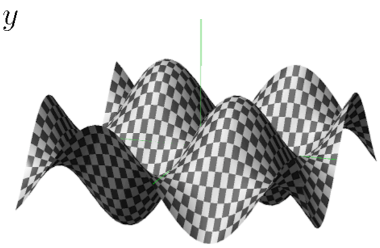
$\boxed{\text{(A)}}$ $\sin x \sin y$

(B) xy

(C) $x + y$

(D) x/y

(E) $x - y$



$\sin x \sin y$ is the only choice that is periodic, as is the graph.

39. A square and a circle have the same area. If the length of the side of the square is tripled and the radius of the circle is tripled, what is the ratio of the area of the new circle to the area of the new square?

(A) $\frac{3}{2}$

(B) π

(C) $\frac{1}{3}\pi$

(D) $\frac{1}{3}$

$\boxed{\text{(E)}}$ 1

Both areas scale with the square of the length (side or radius), so since they were equal areas to begin with and the length was scaled by a factor of 3 in each case then the new areas will also be equal.

40. Some women and some horses (and that's all) are in a stable. In all, there are 22 heads and 72 legs. How many women and how many horses are in the stable?

(A) 8 women and 12 horses

(B) 10 women and 12 horses

(C) 12 women and 10 horses

(D) 14 women and 8 horses

$\boxed{\text{(E)}}$ none of the above

Let p be the number of people, and a the number of animals. The problem can be modeled as a system of linear equations:

$$\begin{aligned} p + a &= 22 \\ 2p + 4a &= 72 \end{aligned}$$

whose solution is $p = 8$ and $a = 14$.