Snow College Mathematics Contest

April 4, 2006 Form: A

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.
2. Go ahead now and fill in the box at the top of your answer sheet. Print your name clearly, put your phone number in the "ID#" blank, spell out your school in the "class" blank, and put your year in school in the "sec" blank. Put your test version (Form A) in the "test no." blank. Also use a #2 (or HB or soft) pencil to bubble in your name on the left side of the answer sheet.
3. This is a two hour examination consisting of 40 multiple choice problems. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is Score $= 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from -40 to 160.
4. In the event of a tie, the person with the largest number of the following five problems correct will be delcared the winner: 40, 34, 25, 3, 20. Any further ties will be broken by a coin toss.
5. When the test begins, bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Use #2 (or soft) pencil. Erase any incorrect answers completely.
6. The sketches that accompany the problems are not necessarily drawn to scale.
7. No calculators are allowed.
8. Do not talk or disrupt other test takers during the exam. Cell phones must be OFF.
9. Please raise your hand if you need scratch paper; a proctor will assist you.
10. The proctors have been advised to answer no questions pertaining to the exam.
11. While we recommend you stay and recheck your answers if you have time, you may leave if you finish early (if you do, turn your answer sheet in and leave quietly). After the two hour time limit is up the proctors will call for your answer sheets. Hand them in promptly.

After the test:

1. You may keep this test booklet.
2. If you RSVP'd to spend time with one of our science departments for lunch, please meet them in the science building; otherwise lunch may be purchased at the Snow College Cafeteria or downtown. In any event, you should plan to be back at the LDS Institute by 1:30 p.m. for the scores and presentation of the awards.
3. The top three scorers from each classification of school will receive full tuition scholarships to Snow College. Other prizes will be awarded to other individuals.
4. Thanks for coming. Your instructors will be happy to work the problems for you, and they will also be given your corrected answer sheets.

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Bubble in the single best answer to each question.

1. When $x = 10$, the expression $\sqrt{1+2+3+x}$ has the value 4. What is the sum of all four integers $x < 10$ for which $\sqrt{1+2+3+x}$ has an integer value?

   (A) -10
   (B) -6
   (C) 6
   (D) 10
   (E) -4

2. A box contains 4 fair coins and 6 biased coins. Whenever a fair coin is flipped, it comes up heads with a probability 0.5. Whenever a biased coin is flipped, it comes up heads with probability 0.8. A coin is randomly chosen from the box and then flipped. What is the probability that it will come up heads?

   (A) 0.6
   (B) 0.64
   (C) 0.68
   (D) 0.72
   (E) 0.76

3. What is the value of the product $(\log_2 3)(\log_3 5)(\log_5 8)$?

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

4. Both x and y are positive real numbers less than 2. Every positive number less than 2 is equally likely to be the value of x; and every positive number less than 2 is equally likely to be the value of y. What is the probability that x and y differ by less than 1?

   (A) 0.20
   (B) 0.25
   (C) 0.50
   (D) 0.65
   (E) 0.75

5. If the lengths of two sides of a right triangle are 3 and 4, what is the least possible length of the third side?

   (A) $\sqrt{7}$
   (B) 3
   (C) 4
   (D) 5
   (E) 7

6. The sides of a triangle are in the ratio 3 : 5 : 9. Which of the following words best describes the triangle?

   (A) obtuse
   (B) scalene
   (C) right
   (D) isosceles
   (E) impossible

7. Let $f(x) = \sqrt{(x-2)^2}$. Compute $\sum_{x=-2}^{x=2} f(2x)$.

(A) -7

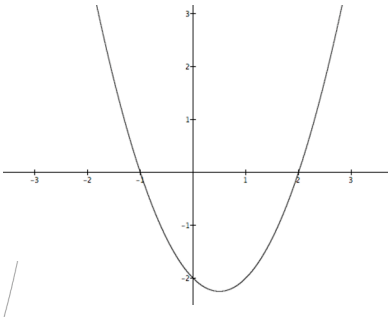
(B) 0

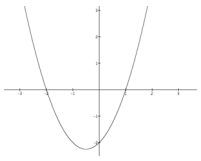
(C) 7

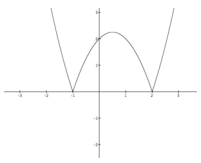
(D) 14

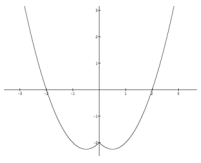
(E) 16

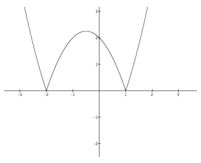
8. Let $f(x)$ be the function whose graph is shown. Which of the following represents the graph of $f(|x|)$?

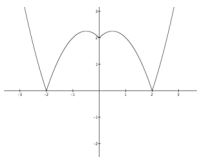


(A) 

(B) 

(C) 

(D) 

(E) 

9. If the operation $\oplus$ is defined for all positive x and y by $x \oplus y = (xy)/(x+y)$, which of the following must be true for positive x, y, and z?

(i) $x \oplus x = x/2$

(ii) $x \oplus y = y \oplus x$

(iii) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

(A) (i) only

(B) (i) and (ii) only

(C) (i) and (iii) only

(D) (ii) and (iii) only

(E) all three

10. On a musical instrument with 4 valves, how many different fingerings are possible?

(A) 8

(B) 10

(C) 12

(D) 16

(E) 20

11. An explorer wishes to cross a barren desert that requires 6 days to cross, but one man can only carry enough food for 4 days. What is the fewest number of other men required to help carry enough food for him to cross (and everyone stay alive)?

(A) 1

(B) 2

(C) 3

(D) 4

(E) it can't be done with any number of extra men

12. In this problem all numbers are written in base 3. What is 21×21?

(A) 1211

(B) 1001

(C) 11111

(D) 1121

(E) 441

13. A square has four corners (or vertices), four edges, and one face. A cube has eight corners, twelve edges, six faces, and one volume. How many edges would a 4-D hypercube (*a.k.a.*, a tesseract) have?

(A) 24

(B) 28

(C) 30

(D) 32

(E) 36

14. Where do the the asymptote of $\frac{x^2+5x+6}{x-2}$ and the line with equation $y = 2x + 3$ cross?

(A) Quadrant I

(B) Quadrant II

(C) Quadrant III

(D) Quadrant IV

(E) On an axis

15. What is $\cot(\frac{\pi}{6})$? (Hint: bisect one angle of an equilateral triangle whose sides are length 2.)

(A) 2

(B) $\frac{\sqrt{3}}{3}$

(C) $\frac{2\sqrt{3}}{3}$

(D) $\frac{\sqrt{3}}{2}$

(E) $\sqrt{3}$

16. The n^{th} triangular number is defined as the sum of the first n whole numbers. (One can arrange dots in a triangular pattern with one dot in the first row, two in the second, etc.) Thus, the 4^{th} triangular number is 10 becuase $10 = 1 + 2 + 3 + 4$. Carl Freidrich Gauss' elementary school class was given the task of adding up the the integers from 1 to 100 in the hopes that it would take the students a lot of time. The teacher was astonished when Gauss immediately came up with what value for the 100^{th} triangular number?

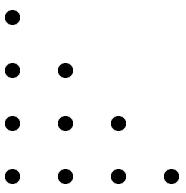
(A) 4900

(B) 4949

(C) 4950

(D) 5000

(E) 5050



17. The left side of the equation is the determinant of the matrix product. The equation

$$\left| \begin{bmatrix} x & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} -2 & x \\ 0 & 0 \end{bmatrix} \right| = 0$$

(A) is satisfied for only one value of x.

(B) is satisfied for two values of x.

(C) is satisfied for no values of x.

(D) is satisfied for an infinite number of values of x.

(E) None of these.

18. What is the equation of the line passing through the point $(1, 7)$ and perpendicular to the line with x-intercept 6 and y-intercept 2?

(A) $y = 3x + 6$

(B) $3x + y = 6$

(C) $-3x + y = 4$

(D) $y = -3x + 4$

(E) $x + 3y = 6$

19. If $f(z) = \frac{z^3+2z}{z+1}$ and $\mathrm{i} = \sqrt{-1}$ then what is $f(\mathrm{i})$?

(A) $\mathrm{i} - 1$

(B) $\mathrm{i} + 1$

(C) $1 - \mathrm{i}$

(D) $\sqrt{2}$

(E) none of the above

20. Two sets are said to have the same cardinality if the elements of one can be put into one-to-one correspondence with the elements of the other. Which of the sets below does not have the same cardinality as at least one of the others? (Hint: think of rules that show the one-to-one correspondences.) (Infinite sets are defined to be ones that can be put into one-to-one correspondence with proper subsets of themselves.)

(A) $\mathbb{N}$, the natural numbers $= \{1, 2, 3, \ldots\}$

(B) $\mathbb{W}$, the whole numbers $= \mathbb{N} + \{0\}$

(C) $\mathbb{Z}$, the integers

(D) $\mathbb{Q}$, the rational numbers

(E) $\mathbb{R}$, the real numbers

21. Given the sequence $\sqrt{3}, \sqrt{6}, 3, 2\sqrt{3}, \ldots$, which term would be $3\sqrt{2}$?

(A) 6^{th}

(B) 7^{th}

(C) 10^{th}

(D) 11^{th}

(E) 12^{th}

22. Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, what is $(g \circ f)(-2)$ where $(g \circ f)$ is the composition of the functions?

(A) -4

(B) 0

(C) 2

(D) 4

(E) 8

23. Long-distance radio navigation for aircraft and ships uses synchronized pulses (which travel at the speed of light) transmitted by two widely separated transmitting stations. The ship or plane uses the differences in arrival times of these pulses to determine its location. Which kind of conic section would best describe the path of such a ship or plane if the differences in pulse arrival times doesn't change over time?

(A) circle

(B) ellipse

(C) parabola

(D) hyperbola

(E) none of the above

24. Given $f(x) = 2x^3 - 3x^2 - 12x + 18$, at what x value does $f(x)$ have a local minimum?

(A) -1

(B) 2

(C) 3

(D) 6

(E) none of the above

25. The binomial coefficient (read "n choose r") is defined by

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

What is $\binom{12}{2}$?

(A) 60

(B) 66

(C) 120

(D) 47 900 160

(E) 239 500 800

26. What is the output of the following computer program?

```
10 for i = 1 to 3
20 for j = 1 to i
30 print i*j
40 next j
50 next i
```

(A) 1, 2, 4, 3, 6, 9

(B) 1, 2, 3, 1, 2, 3

(C) 1, 2, 2, 3, 3, 3

(D) 1, 2, 3, 4, 5, 6, 7, 8, 9

(E) 1, 2, 3, 1, 2, 3, 1, 2, 3

27. What curve is represented by the parametric equations?

$$\left.\begin{aligned} x &= 2\sin t \\ y &= 3\cos t \end{aligned}\right\} \text{for } t \text{ in } [0, 2\pi]$$

(A) circle

(B) hyperbola, horizontal transverse axis

(C) hyperbola, vertical transverse axis

(D) ellipse, horizontal major axis

(E) ellipse, vertical major axis

28. Two cans (right cylinders) have the same volume. The height of one can is triple the height of the other. If the radius of the narrower can is 12 cm, what is the radius of the wider can?

(A) $2\sqrt{12}$ cm

(B) $12\sqrt{3}$ cm

(C) $6\sqrt{3}$ cm

(D) $3\sqrt[3]{12}$ cm

(E) $3\sqrt{12}$ cm

29. What is the sum of the distances AD and BD in the figure?

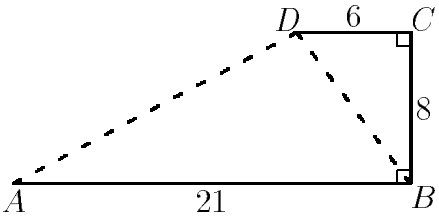
(A) 27

(B) 28

(C) 29

(D) 30

(E) 31



30. The six points A, B, C, D, E, F are equally spaced along the circumference of a circle of radius 1. Line segments $\overline{AC}, \overline{CE}, \overline{AE}, \overline{BD}, \overline{DF}$, and $\overline{BF}$ are drawn. What is the area of the six-pointed star-shaped region?

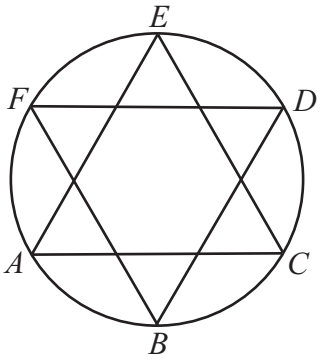
(A) $\frac{2\sqrt{6}}{3}$

(B) $\sqrt{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{3\sqrt{2}}{2}$

(E) $\frac{3\pi}{4}$



31. A drawer has 6 red socks and 6 white socks. If you reach in the drawer and randomly take out two socks, what is the chance (*i*.e., probabilitiy) that the two socks will match in color?

(A) $\frac{5}{12}$

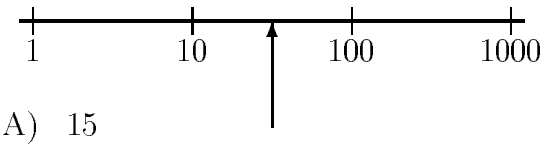
(B) $\frac{1}{2}$

(C) $\frac{2}{5}$

(D) $\frac{3}{7}$

(E) $\frac{5}{11}$

32. The diagram shows the scale on a measuring device. What is the approximate reading at the arrow?



(A) 15
(B) 32
(C) 50
(D) 55
(E) 75

33. What is the sum of the exponents of the prime factorization of 200?

(A) 2
(B) 4
(C) 5
(D) 7
(E) 8

34. By using the answers to the following questions, Chris determines Terry's secret number. What is Terry's secret number?

- Is it a factor of 30? Yes
- Is it a prime number? No
- Is it a multiple of 3? No
- Is it less than 3? No

(A) 3
(B) 5
(C) 6
(D) 10
(E) 15

35. If you start with 128 g of carbon-14 (^{14}C), whose half-life is 5700 years, how much ^{14}C will be left after 28 500 years?

(A) 128 g
(B) 64 g
(C) 32 g
(D) 4 g
(E) Approximately zero

36. What quadrilateral can be divided into three equilateral triangles?

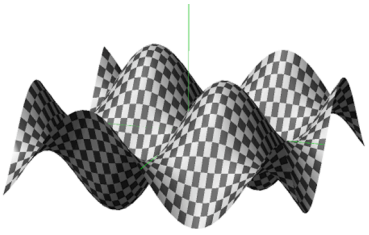
(A) square
(B) rectangle
(C) rhombus
(D) parallelogram
(E) isosceles trapezoid

37. Given $\omega^3 = 1$ and $\omega \neq 1$, what is $\omega(\omega+1)$?

(A) i
(B) $-$i
(C) 1
(D) -1
(E) $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}$

38. Which function of x and y is represented by the graph?

(A) $\sin x \sin y$
(B) xy
(C) $x + y$
(D) x/y
(E) $x - y$



39. A square and a circle have the same area. If the length of the side of the square is tripled and the radius of the circle is tripled, what is the ratio of the area of the new circle to the area of the new square?

(A) $\frac{3}{2}$

(B) π

(C) $\frac{1}{3}\pi$

(D) $\frac{1}{3}$

(E) 1

40. Some women and some horses (and that's all) are in a stable. In all, there are 22 heads and 72 legs. How many women and how many horses are in the stable?

(A) 8 women and 12 horses

(B) 10 women and 12 horses

(C) 12 women and 10 horses

(D) 14 women and 8 horses

(E) none of the above