Snow College Mathematics Contest

April 5, 2005 Form: A

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.
2. Go ahead now and fill in the box at the top of your answer sheet. Print your name clearly, put your phone number in the "ID#" blank, spell out your school in the "class" blank, and put your year in school in the "sec" blank. Put your test version (Form A) in the "test no." blank. Also use a #2 (or HB or soft) pencil to bubble in your name on the left side of the answer sheet. Also in the lower left corner where it says Identification Number, bubble in your school code in the first two digits, leave the next three blank, and put the last four digits of your social security number in the last four bubbles. Having bubbles in the last four digits is very important, so if you don't know your SSN then just bubble in four random digits.
3. This is a two hour examination. Do not talk or disrupt other test takers during the exam.
4. The test consists of 40 multiple choice problems. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is Score $= 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from -40 to 160.
5. In the event of a tie, the person with the largest number of the following five problems correct will be delcared the winner: 36, 11, 39, 32, 24. Any further ties will be broken by a coin toss.
6. When the test begins, bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Use #2 (or soft) pencil. Erase any incorrect answers completely.
7. The sketches that accompany the problems are not necessarily drawn to scale.
8. No calculators are allowed.
9. Please raise your hand if you need scratch paper; a proctor will assist you.
10. The proctors have been advised to answer no questions pertaining to the exam.
11. While we recommend you stay and recheck your answers if you have time, you may leave if you finish early (if you do, turn your answer sheet in and leave quietly). After the two hour time limit is up the proctors will call for your answer sheets. Hand them in promptly.

After the test:

1. You may keep this test booklet.
2. Lunch may be purchased at the Snow College Cafeteria or downtown. However, you should plan to be back at the LDS Institute by 1:30 p.m. for the presentation of the awards and scores.
3. The top three scorers from each classification of school will receive full tuition scholarships to Snow College. Other prizes will be awarded to other individuals.
4. Thanks for coming. Your instructors will be happy to work the problems for you, and they will also be given your corrected answer sheets.

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April 5, 2005 Form: A

Bubble in the single best answer to each question.

1. What is the sum of the squares of all solutions for $x^2 = \sqrt{5x^2 - 4}$?

   (A) 1
   (B) 2
   (C) 5
   (D) 6
   (E) 10

2. A pup is worth a pooch and a mutt. A pup and a pooch are worth one bird dog. Two bird dogs are worth three mutts. How many pooches is a pup worth?

   (A) 4
   (B) 6
   (C) 2
   (D) 5
   (E) Not enough information

3. Which of the following is an asymptote of the function $f(x) = \frac{x^2-2x+1}{2x^2-1}$?

   (A) $x = 0$
   (B) $x = 2$
   (C) $x = \frac{1}{2}$
   (D) $y = 2$
   (E) $y = \frac{1}{2}$

4. If a square has a diagonal of length b, then what is the area of the square?

   (A) $b^2/4$
   (B) $b^2/2$
   (C) b^2
   (D) $\sqrt{3}b^2/2$
   (E) Not enough information

5. One root of $mx^2 + 8x + 4 = 0$ is three times the other root. What is the value of m?

   (A) -5
   (B) 5
   (C) $-\frac{28}{9}$
   (D) -3
   (E) 3

6. If two real numbers x and y are such that xy, $\frac{x}{y}$, and $x + y$ are all equal, then what is $x - y$?

   (A) $\frac{-3}{2}$
   (B) $\frac{3}{2}$
   (C) $\frac{1}{2}$
   (D) $\frac{-1}{2}$
   (E) None of the above

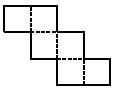
7. If $f(x) = \frac{x^6+x^4+2x^2}{x+1}$ and $\mathrm{i} = \sqrt{-1}$ then what is $f(\mathrm{i})$?

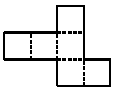
   (A) $\mathrm{i} - 1$
   (B) $\mathrm{i} + 1$
   (C) $1 - \mathrm{i}$
   (D) $\sqrt{2}$
   (E) None of the above

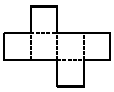
8. If the lengths of the sides of a right triangle form an arithmetic sequence, then what is the ratio of these sides?

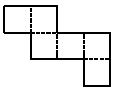
   (A) 1:1:1
   (B) 1:2:3
   (C) 3:4:5
   (D) 1:4:9
   (E) Not enough information

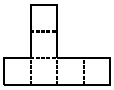
9. Which of the following patterns below CANNOT be folded along the dashed lines to form a cube?

(A) 

(B) 

(C) 

(D) 

(E) 

10. $1 \diamondsuit 3 = 5, \quad 6 \diamondsuit 9 = 21, \quad 8 \diamondsuit 2 = 18.$
Find $11 \diamondsuit 20$.

(A) 221

(B) 55

(C) 51

(D) 66

(E) 42

11. Consider the geometric sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^n}, \ldots$. If S_n is the n^{th} partial sum, what is the value of $\lim_{n\to\infty} S_n$?

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) ∞

(E) $\frac{1}{2^\infty}$

12. Which is equal to $(a^{-1} + b^{-1})^{-1}$?

(A) $\frac{a+b}{ab}$

(B) $\frac{ab}{a+b}$

(C) $a + b$

(D) $\frac{1}{a+b}$

(E) $\frac{1}{a^2+b^2}$

13. What is the equation of the line passing through the point $(4, 2)$ and perpendicular to the line with x-intercept 2 and y-intercept -1?

(A) $y = 2x + 10$

(B) $x + 2y = 8$

(C) $2x + y = 10$

(D) $y = -2x + 8$

(E) $x + 2y = 10$

14. Given the sequence $\sqrt{2}, \sqrt{5}, 2\sqrt{2}, \sqrt{11}, \ldots$, which term would be $2\sqrt{5}$?

(A) 6^{th}

(B) 7^{th}

(C) 10^{th}

(D) 11^{th}

(E) 12^{th}

15. A cube and a sphere have the same volume. If the length of the side of the cube is doubled and the radius of the sphere is doubled, what is the ratio of the volume of the new sphere to the volume of the new cube?

(A) $\frac{3}{4}$

(B) π

(C) $\frac{4}{3}\pi$

(D) $\frac{4}{3}$

(E) 1

16. Solve the following equation for r.

$$\frac{F}{f} = \frac{R + r}{r}$$

(A) $\frac{fR}{F-f}$

(B) $\frac{F-f}{fR}$

(C) $\frac{Fr-fR}{f}$

(D) $\frac{f}{Fr-fR}$

(E) $\frac{FR+fr}{F}$

17. Let $2^a = 3$, $2^b = 6$, and $2^c = 12$. Then which of the following is true?

(A) a, b, c is an arithmetic sequence, but *not* a geometric sequence.

(B) a, b, c is a geometric sequence, but *not* an arithmetic sequence.

(C) a, b, c is both an arithmetic sequence, *and* a geometric sequence.

(D) a, b, c is neither an arithmetic sequence, *nor* a geometric sequence.

(E) Not enough information

18. Let the xy-plane be divided into nine regions by the lines $x = \pm 2$, $y = \pm 2$. In how many of these nine regions does the graph of $y^2 = \frac{4x^2}{x^2-1}$ have any points?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

19. What is the maximum number of acute interior angles in a regular polygon?

(A) 2

(B) 3

(C) 4

(D) 5

(E) None of the above

20. The dimensions, in inches, of a rectangular box are all positive integers; and the volume of the box is 2002 cubic inches. Find the minimum possible sum, in inches, of the three dimensions.

(A) 38

(B) 42

(C) 104

(D) 32

(E) 40

21. Ten people are in a room. Of the ten, at least one person is a student, and in any group of 3, at least 1 is not a student. How many students are in the room?

(A) exactly 1

(B) exactly 2

(C) 1 or 2

(D) exactly 3

(E) 3 or 4

22. What is the smallest interval for k (a real number) such that $x^2 + ky^2 = 2$ represents an ellipse whose foci lie on the y-axis?

(A) $(0, \infty)$

(B) $(0, 2)$

(C) $(1, \infty)$

(D) $(-\infty, \infty)$

(E) $(0, 1)$

23. Three-digit numbers with no repeating digits are to be formed by selecting digits from $\{1, 2, 3, 4, 5\}$. How many even numbers can be formed in this manner?

(A) 24

(B) 30

(C) 40

(D) 50

(E) 60

24. There are six cards colored red, yellow, or blue on one side. The other side of each card has one of the symbols $\circledcirc$, $\bowtie$, or $\divideontimes$ on it. Consider the statement: "Every yellow card has a $\divideontimes$ on the other side." To prove or disprove the statement, which of the following cards must be turned over and checked?

red	$\circledcirc$	yellow
1	2	3
$\bowtie$	blue	$\divideontimes$
4	5	6

(A) card 3 only

(B) cards 3 and 6 only

(C) cards 2, 3, and 4 only

(D) cards 2, 3, and 6 only

(E) cards 2, 3, 4, and 6 only

25. Which of the following does NOT equal -4? ($\mathrm{i} = \sqrt{-1}$)

(A) -2^2

(B) $(\frac{-1}{4})^{-1}$

(C) $(2\mathrm{i})^2$

(D) $\log_{16}(\frac{1}{2})$

(E) All equal -4

26. The midpoints of the sides of $\triangle ABC$ are $(1,1)$, $(2,3)$, and $(5,1)$. Find the area of $\triangle ABC$.

(A) 16

(B) 18

(C) 20

(D) 22

(E) Not enough information

27. If $y = \frac{1}{2}\sin 2x + \sin x$, which of the following statements is true about y'?

(A) It is an odd function that only has one minimum.

(B) It is an odd function that only has one maximum.

(C) It is an even function that has both maxima and minima.

(D) It is an even function that has no extrema.

(E) It is neither an odd nor an even function.

28. The cross section through the axis of symmetry of a cylinder is a square $\square ABCD$ with sides of 5 cm. What is the shortest distance from A to C along the surface of the cylinder?

(A) 10 cm

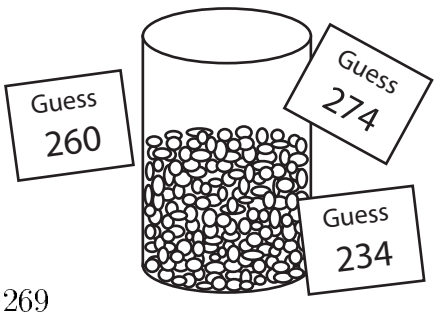
(B) $\frac{5}{2}\sqrt{\pi^2+4}$ cm

(C) $5\sqrt{2}$ cm

(D) $5\sqrt{\pi^2+1}$ cm

(E) 5π cm

29. One guess is off by nine, another guess is off by seventeen, and the other guess is off by thirty-one. How many jelly beans are in the jar?



(A) 269

(B) 251

(C) 265

(D) 245

(E) 243

30. Where do the lines with equations $\frac{x}{2}+\frac{y}{3}=1$ and $6x+2y=13$ intersect?

(A) Quadrant I

(B) Quadrant II

(C) Quadrant III

(D) Quadrant IV

(E) On an axis

31. If $f(x)=x^2$, then what is $\frac{f(x+1)-f(x)}{2}$?

(A) $x+1$

(B) $\frac{1}{2}$

(C) $x+\frac{1}{2}$

(D) $x+\frac{1}{4}$

(E) None of the above

32. Let $A=\{1,2,3,4\}$, $B=\{1,5,6,7,8\}$. How many distinct points in the xy-plane can be formed by choosing one coordinate from each of the sets?

(A) 19

(B) 20

(C) 39

(D) 40

(E) None of the above

33. Let a,b,c be the lengths of three sides of $\triangle ABC$ with A,B,C the respective angles. If $\sin A\cdot\cos A=0$ and $a=2c\cos B$, what is the shape of $\triangle ABC$?

(A) equilateral triangle

(B) right isosceles triangle

(C) right triangle with unequal legs

(D) non-right isosceles triangle

(E) None of these

34. The graph in the xy-plane of which function below does NOT intersect the y-axis?

(A) $y=\tan x$

(B) $y=\cos x$

(C) $y=\sec x$

(D) $y=\csc x$

(E) None of the above

35. All integers greater than 1 are arranged in columns as shown.

	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
	18	19	20	21
	⋮			

In which column will the integer 640 fall?

(A) 4th

(B) 1st

(C) 2nd

(D) 3rd

(E) 5th

36. If one-fifth of two-fifths more than x is three-fifths less than four-fifths of x, then what is two-fifths of x?

(A) $\frac{17}{15}$

(B) $\frac{2}{25}$

(C) $\frac{13}{25}$

(D) $\frac{34}{75}$

(E) $\frac{26}{125}$

37. What is the remainder when $5x^{56} - 3x^{47} - x^{20} - 4x^5 - 2$ is divided by $x + 1$?

(A) -5

(B) -2

(C) 1

(D) 9

(E) None of these

38. Simplify.

$$\frac{8^{50} + 8^{49}}{8^8 + 8^7}$$

(A) $\frac{99}{15}$

(B) 8^7

(C) 7

(D) 8^{42}

(E) 8^{54}

39. Water in a vertical right cylinder has a depth of 6 cm. The cylinder has a radius of 2 cm. If we pour all of the water into an upside down cone whose cross section is an equilateral triangle, what will the depth of the water be?

(A) $2\sqrt[3]{18}$ cm

(B) 6 cm

(C) $6\sqrt{3}$ cm

(D) $3\sqrt[3]{12}$ cm

(E) Not enough information

40. If α and β are the solutions of $2x^2+3x-5 = 0$ then what is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{3}{2}$

(D) $\frac{2}{3}$

(E) None of the above