Snow College Mathematics Contest   April 6, 2004   Form: A

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.

2. Go ahead now and fill in the box at the top of your answer sheet. Print your name clearly, put your phone number in the "ID#" blank, spell out your school in the "class" blank, and put your year in school in the "sec" blank. Put your test version (Form A) in the "test no." blank. Also use a #2 (or HB or soft) pencil to bubble in your name on the left side of the answer sheet. Also in the lower left corner where it says Identification Number, bubble in your school code in the first two digits, leave the next three blank, and put the last four digits of your social security number in the last four bubbles. Having bubbles in the last four digits is very important, so if you don't know your SSN then just bubble in four random digits.

3. This is a two hour examination. Do not talk or disrupt other test takers during the exam.

4. The test consists of 40 multiple choice problems. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is Score $= 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from -40 to 160.

5. In the event of a tie, the person with the largest number of the following five problems correct will be delcared the winner: 22, 28, 13, 20, 24. Any further ties will be broken by a coin toss.

6. When the test begins, bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Use #2 (or soft) pencil. Erase any incorrect answers completely.

7. The sketches that accompany the problems are not necessarily drawn to scale.

8. No calculators are allowed.

9. Please raise your hand if you need scratch paper; a proctor will assist you.

10. The proctors have been advised to answer no questions pertaining to the exam.

11. While we recommend you stay and recheck your answers if you have time, you may leave if you finish early (if you do, turn your answer sheet in and leave quietly). After the two hour time limit is up the proctors will call for your answer sheets. Hand them in promptly.


After the test:

1. You may keep this test booklet.

2. Lunch may be purchased at the Snow College Cafeteria or downtown. However, you should plan to be back at the LDS Institute by 1:30 p.m. for the presentation of the awards and scores.

3. The top three scorers from each classification of school will receive full tuition scholarships to Snow College. Other prizes will be awarded to other individuals.

4. Thanks for coming. Your instructors will be happy to work the problems for you, and they will also be given your corrected answer sheets.

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Bubble in the single best answer to each question.

1. Of the given sets, which includes all values of x which will satisfy $2x - 3 > 7 - x$?

    (A) $\{x \mid x > 4\}$

    (B) $\{x \mid x < 10/3\}$

    (C) $\{x \mid x = 10/3\}$

    (D) $\{x \mid x > 10/3\}$

    (E) $\{x \mid x < 0\}$

2. In the diagram, if points A, B, C are points of tangency, then what is x?
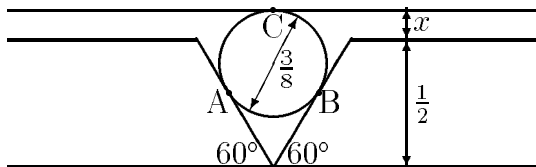
    (A) 3/16

    (B) 1/8

    (C) 1/32

    (D) 3/32

    (E) 1/16

3. A boy born in the first half of the nineteenth century was n years old in the year n^2. In what year was he born?

    (A) 1849

    (B) 1825

    (C) 1812

    (D) 1836

    (E) 1806

4. A boat goes 15 mph in still water. In a stream with a current of 5 mph it travels a certain distance downstream and returns. What is the ratio of the average speed for the round trip to the speed in still water?

    (A) 5/4

    (B) 1/1

    (C) 8/9

    (D) 7/8

    (E) 9/8

5. Each of two angles of a triangle is 60° and the included side is 4 inches. What is the area of the triangle in square inches?

    (A) $8\sqrt{3}$

    (B) 8

    (C) $4\sqrt{3}$

    (D) 4

    (E) $2\sqrt{3}$

6. If the radius of a circle is a rational number, its area is given by a number which is

    (A) rational.

    (B) irrational.

    (C) integral.

    (D) a perfect square.

    (E) none of the above.

7. If the discriminant of $ax^2 + 2bx + c = 0$ is zero, then another true statement about a, b, and c is that

   (A) they are terms of an arithmetic progression.

   (B) they are terms of a geometric progression.

   (C) they are unequal.

   (D) they are all negative numbers.

   (E) only b is negative and a and c are positive.

8. A six digit number is formed by repeating a three digit number, e.g., $256\,256$ or $678\,678$. Every number of this form is divisible by

   (A) 7 only.

   (B) 11 only.

   (C) 13 only.

   (D) 101.

   (E) 1001.

9. At 2:15 o'clock, what is the angle between the hour and minute hands of a clock?

   (A) $30°$

   (B) $5°$

   (C) $22\frac{1}{2}°$

   (D) $17\frac{1}{2}°$

   (E) $28°$

10. Through a point inside a triangle, three lines are drawn from the vertices to the opposite sides forming six triangular sections. Then

    (A) the triangles are similar in opposite pairs.

    (B) the triangles are congruent in opposite pairs.

    (C) the triangles are equal in area in opposite pairs.

    (D) three similar quadrilaterals are formed.

    (E) none of the above.

11. A rise of 600 ft is required to get a railroad up a mountain. The grade can be kept down by lengthening the track and curving it around the mountain peak. What additional length of track is required to reduce the grade from 3% to 2%?

    (A) 10000 ft

    (B) 20000 ft

    (C) 30000 ft

    (D) 12000 ft

    (E) None of these.

12. What is the value of $10^{\log 7}$?

    (A) 7

    (B) 1

    (C) 10

    (D) $\log 7$

    (E) $\log_7 10$

13. If $a^x = c^q = b$ and $c^y = a^z = d$ then

    (A) $xy = qz$.

    (B) $\frac{x}{y} = \frac{q}{z}$.

    (C) $x + y = q + z$.

    (D) $x - y = q - z$.

    (E) $x^y = q^z$.

14. Mae sold a house and a store for $12000 each. The house was sold at a loss of 20% of its cost, and the store at a gain of 20% of its cost. The entire transaction resulted in

(A) no loss or gain.

(B) a loss of $1000.

(C) a gain of $1000.

(D) a gain of $2000.

(E) none of these.

15. If the expression $\begin{vmatrix} a & c \\ d & b \end{vmatrix}$ has a value $ab - cd$ for all values of a, b, c, and d, then the equation $\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 3$

(A) is satisfied for only one value of x.

(B) is satisfied for two values of x.

(C) is satisfied for no values of x.

(D) is satisfied for an infinite number of values of x.

(E) None of these.

16. What is the least value of the function $f(x) = ax^2 + bx + c \ (a > 0)$?

(A) $-\frac{b}{a}$

(B) $-\frac{b}{2a}$

(C) $b^2 - 4ac$

(D) $\frac{4ac - b^2}{4a}$

(E) None of these.

17. A privateer discovers a merchantwoman 10 miles leeward at 11:45 a.m. and with a good breeze bears down upon her at 11 mph, while the merchantwoman can only make 8 mph in her attempt to escape. After a two hour chase, the topsail of the privateer is torn away; he can now travel 17 miles in the same time it takes the merchantwoman to travel 15 miles. At what time will the privateer overtake the merchantwoman?

(A) 3:45 p.m.

(B) 3:30 p.m.

(C) 5:00 p.m.

(D) 2:45 p.m.

(E) 5:30 p.m.

18. A manufacturer built a machine which will address 500 envelopes in 8 minutes. He wishes to build another machine so that when both are operating together they will address 500 envelopes in 2 minutes. What is the equation used to find how many minutes x it would require the second machine to address 500 envelopes alone?

(A) $8 - x = 2$

(B) $\frac{1}{8} + \frac{1}{x} = \frac{1}{2}$

(C) $\frac{500}{8} + \frac{500}{x} = 500$

(D) $\frac{x}{2} + \frac{2}{8} = 1$

(E) None of these

19. The graph of $y = \log x$

(A) cuts the y-axis.

(B) cuts all lines perpendicular to the x-axis.

(C) cuts the x-axis.

(D) cuts neither axis.

(E) cuts all circles centered on the origin.

20. What is the sum of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$?

    (A) $\frac{5}{2}$

    (B) 2

    (C) 1.9

    (D) 1.75

    (E) None of these

21. If you are given $\log 8 = 0.9031$ and $\log 9 = 0.9542$ then which is the only logarithm of the following that cannot be found without the use of tables or a calculator?

    (A) $\log 17$

    (B) $\log \frac{5}{4}$

    (C) $\log 15$

    (D) $\log 600$

    (E) $\log 0.4$

22. Which of the five statements negates the statement "All men are good golfers."?

    (A) All women are good golfers.

    (B) Some women are good golfers.

    (C) No men are good golfers.

    (D) All men are bad golfers.

    (E) At least one man is a bad golfer.

23. What is the inverse of the function?
    $f(x) = \frac{4x}{2x-1} \quad (x \neq \frac{1}{2})$

    (A) $f^{-1}(x) = \frac{2x}{2x-1} \quad (x \neq \frac{1}{2})$

    (B) $f^{-1}(x) = \frac{x}{x-4} \quad (x \neq 4)$

    (C) $f^{-1}(x) = \frac{x}{2x-4} \quad (x \neq 2)$

    (D) $f^{-1}(x) = \frac{x}{2x-2} \quad (x \neq 1)$

    (E) $f^{-1}(x) = \frac{2x}{x-4} \quad (x \neq 4)$

24. If $f(x) = 3x + 2$ and $g(x) = 4x - 1$, what is $(f \circ g)(2)$?

    (A) $\frac{7}{8}$

    (B) $\frac{8}{7}$

    (C) 31

    (D) 23

    (E) 15

25. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{2, 3, 5, 7\}$. What is $A \cap (B \cup C)$?

    (A) $\{1, 2, 3, 4\}$

    (B) $\{3, 4, 5\}$

    (C) $\{2, 3, 4\}$

    (D) $\{2, 3, 4, 5, 7\}$

    (E) $\emptyset$

26. What is $\sec(\arcsin \frac{4}{5})$?

    (A) $\frac{4}{5}$

    (B) $\frac{4}{3}$

    (C) $\frac{5}{4}$

    (D) $\frac{5}{3}$

    (E) $\frac{3}{5}$

27. What is the area of the triangle with vertices $(1, 0), (2, 2)$, and $(4, 3)$? (*Hint:* determinant.)

    (A) $\frac{3}{2}$

    (B) $\frac{2}{3}$

    (C) $\frac{4}{3}$

    (D) $-\frac{2}{3}$

    (E) $\frac{7}{5}$

28. Simplify.

$$\frac{4i}{-4+2i}$$

(A) $\frac{2}{5} - \frac{4}{5}i$

(B) $\frac{4}{5} - \frac{2}{5}i$

(C) $\frac{2}{5} - \frac{4}{6}i$

(D) $\frac{4}{5} - \frac{2}{6}i$

(E) $\frac{2}{6} - \frac{4}{6}i$

29. Which values of a satisfy the equation $\log(a^2 - 15a) = 2$?

(A) $\frac{15 \pm \sqrt{233}}{2}$

(B) $20, -5$

(C) $\frac{15 \pm \sqrt{305}}{2}$

(D) ± 20

(E) None of these

30. If $\triangle ABC$ is inscribed in a semicircle whose diameter is $\overline{AB}$, then $\overline{AC} + \overline{BC}$ must be

(A) equal to $\overline{AB}$.

(B) equal to $\overline{AB}\sqrt{2}$.

(C) $\geq \overline{AB}\sqrt{2}$

(D) $\leq \overline{AB}\sqrt{2}$

(E) $\overline{AB}^2$

31. The relationship between graphs of $y = \sin x$ and $y = \cos x$ is best described by which of the following?

(A) They are inverses of each other.

(B) They are reflections of each other.

(C) One is a dilation of the other.

(D) They are symmetric about the line $y = x$.

(E) None of these.

32. Tom, Dick, and Harry started out on a 100 mile journey. Tom and Harry went by automobile at the rate of 25 mph, while Dick walked at the rate of 5 mph. After a certain distance, Harry got off and walked on at 5 mph, while Tom went back for Dick and got him to the destination at the same time Harry arrived. What was the number of hours required for the trip?

(A) 5

(B) 6

(C) 7

(D) 8

(E) None of these
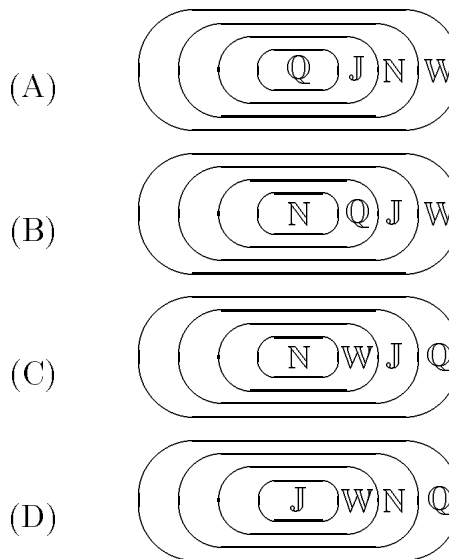
33. Which Venn diagram represents the relationship between the given sets of numbers?

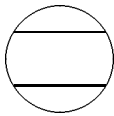$\mathbb{N}$ = Natural Numbers

$\mathbb{Q}$ = Rational Numbers

$\mathbb{J}$ = Integers

$\mathbb{W}$ = Whole Numbers

(A)

(B)

(C)

(D)

(E) None of these

34. Two equal parallel chords are drawn 8 inches apart in a circle of radius 8 inches. What is the area in square inches of the part of the circle that lies between the chords?

(A) $21\frac{1}{3}\pi - 32\sqrt{3}$

(B) $32\sqrt{3} + 21\frac{1}{3}\pi$

(C) $32\sqrt{3} + 42\frac{2}{3}\pi$

(D) $16\sqrt{3} + 42\frac{2}{3}\pi$

(E) $42\frac{2}{3}\pi$

35. Two cyclists, k miles apart, and starting at the same time, would be together in r hours if they traveled in the same direction, but would pass each other in t hours if they traveled in opposite directions. What is the ratio of the speed of the faster cyclist to that of the slower?

(A) $\frac{r+t}{r-t}$

(B) $\frac{r}{r-t}$

(C) $\frac{r+t}{r}$

(D) $\frac{r}{t}$

(E) $\frac{r+k}{t-k}$

36. For $f(x)$ to be continuous, what value must we assign $f(x)$ at $x = -1$ if $f(x) = \frac{x^3+1}{x^2-1}$ for $x \neq -1$?

(A) -2

(B) 0

(C) $\frac{3}{2}$

(D) ∞

(E) $\frac{-3}{2}$

37. If a two digit integer is k times the sum of its digits, the number formed by interchanging the digits is the sum of the digits multiplied by

(A) $(9 - k)$.

(B) $(10 - k)$.

(C) $(11 - k)$.

(D) $(k - 1)$.

(E) $(k + 1)$.

38. What are the roots of $x(x^2 + 8x + 16)(4 - x) = 0$?

(A) 0

(B) $0, 4$

(C) $0, 4, -4$

(D) $0, 4, -4, -4$

(E) None of these

39. What is the number of ounces of water needed to reduce 9 ounces of shaving lotion containing 50% alcohol to a lotion containing 30% alcohol ?

(A) 3

(B) 4

(C) 5

(D) 6

(E) 7

40. A girls' camp is located 300 rods from a straight road. On this road a boys' camp is located 500 rods from the girls' camp. It is desired to build a canteen on the road exactly the same distance from each camp. (Equal rights!) What will that equal distance from each camp be?

(A) 400 rods

(B) 250 rods

(C) 87.5 rods

(D) 200 rods

(E) None of these.