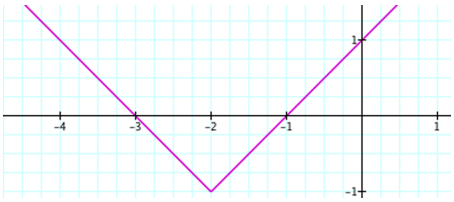
Snow College Jr. Mathematics Contest

key

April 5, 2011 **Junior Division: Grades 7–9** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Which equation best represents the graph?



(A) $y = |x-2|-1$

(B) $y = \sqrt{x^2}-1$

(C) $y = (|x|-2)-1$

(D) $y = |x+2|-1$

(E) $y = \sqrt{(x-2)^2}-1$

SOLN Shift the graph of $y = |x|$ *left* 2 and down 1. $y = \sqrt{(x+2)^2}-1$ also produces the same graph. □

2. In music an interval of a perfect fifth is comprised of two notes whose frequencies have a ratio of 3 to 2. If middle C has a frequency of 260 Hz what is the frequency of the G a perfect fifth above it?

(A) 130 Hz

(B) 173 Hz

(C) 390 Hz

(D) 520 Hz

(E) 780 Hz

SOLN $\frac{3}{2}(260\,\text{Hz}) = 390\,\text{Hz}$ □

3. A collection of coins is made up of an equal number of pennies, nickels, dimes, and quarters. What is the largest possible value of the collection which is less than \$2.00?

(A) \$1.64

(B) \$1.78

(C) \$1.86

(D) \$1.89

(E) \$1.99

SOLN Let A be the number of each coin. Then A is the largest integer that satisfies $A\cdot 1 + A\cdot 5 + A\cdot 10 + A\cdot 25 < 200$ or $41A < 200 \implies A = \lfloor \frac{200}{41} \rfloor = \lfloor 4.878 \rfloor = 4$. Value is $41A = 164$. □

4. How many of the integers from 1 to 100 inclusive do **NOT** contain the digit 7?

(A) 19

(B) 20

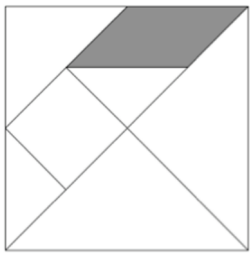
(C) 80

(D) 81

(E) 90

SOLN 1 through 100 is 100 numbers. There are 10 numbers ending in 7 (namely 7, 17, 27, ..., 97). There are 10 numbers that start with 7 (namely 70, 71, 72, ..., 79). However, 77 is in both sets so there are 19 numbers that have at least one 7. Therefore, there are $100-19 = 81$ numbers that do not contain the digit 7. □

5. If a dart lands at random on a square tangram dartboard, what is the probability of its landing in the shaded parallelogram? (The triangles are all isosceles.)



(A) 1/20
(B) 1/10
(C) 1/8
(D) 1/5
(E) 1/4

SOLN Soln 1: You can fit four of the smallest triangles in one of the biggest triangles, and four of the biggest triangles in the whole square. The shaded parallelogram is made of two of the smallest triangles, so the ratio of areas is $\frac{2}{16} = \frac{1}{8}$.
Soln 2: Call the length of a side of the large square s. The area of the parallelogram is $A = bh$ where $b = \frac{1}{2}s$ and $h = \frac{1}{4}s$. The ratio of areas is

$$\frac{A_{\text{para}}}{A_{\text{square}}} = \frac{bh}{s^2} = \frac{(\frac{1}{2}s)(\frac{1}{4}s)}{s^2} = \frac{1}{8} \quad \square$$

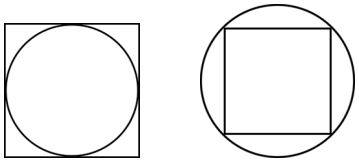
6. Find the product of the matrices.

$$\begin{bmatrix} 4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

(A) 425×316
(B) $\begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} \begin{bmatrix} 5 & 2 & 4 \end{bmatrix}$
(C) [264]
(D) [41]
(E) [44]

SOLN $(4 \times 3) + (2 \times 1) + (5 \times 6) = 44$
The product of a $m \times n$ matrix and an $n \times p$ matrix is a $m \times p$ matrix. When m and p are both 1 (as in this case) we can write a 1×1 matrix as a scalar without square brackets. $\square$

7. Which is a better (tighter) fit: a round peg in a square hole, or a square peg in a round hole? (A tighter fit fills up more of the hole.)



[(A)] round peg in a square hole

(B) square peg in a round hole

(C) both are equally tight

(D) need to know the length of the side of the square

(E) need to know the radius of the circle

[SOLN] The larger the ratio of the cross-sectional area of the peg to the area of the hole, the tighter the fit.

Round peg: $\frac{\pi r^2}{(2r)^2} = \frac{\pi}{4}$ Square peg: $\frac{\left(\frac{2}{\sqrt{2}}r\right)^2}{\pi r^2} = \frac{2}{\pi}$

$\frac{\pi}{4} > \frac{2}{\pi}$ because $\frac{\pi}{4} > \frac{3}{4} > \frac{2}{3} > \frac{2}{\pi}$ □

8. Steve is 66 inches tall and Mike is 72 inches tall. How tall is Jacob if the average height of the three is 71 inches?

(A) 70 inches

(B) 72 inches

[(C)] 75 inches

(D) 76 inches

(E) 78 inches

[SOLN] $\frac{66+72+h}{3} = 71 \implies h = 75$ □

9. A popular dice game is called craps. In it you roll two standard six-sided dice and add the numbers showing on the top faces. What is the probability of rolling a sum of either 7 or 11 (called "throwing craps")?

(A) $\frac{1}{9}$

[(B)] $\frac{2}{9}$

(C) $\frac{1}{6}$

(D) $\frac{7}{36}$

(E) $\frac{6}{7}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

[SOLN] Since they are mutually exclusive, $P(7 \text{ or } 11) = P(7)+P(11) = \frac{6}{36}+\frac{2}{36} = \frac{8}{36} = \frac{2}{9}$. □

10. Flip a fair coin. Go 2 for heads, 1 for tails.

Start		Go back 2 spaces	End

What is the probability of reaching End in exactly 3 turns?

[(A)] $\frac{1}{8}$ HTH

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

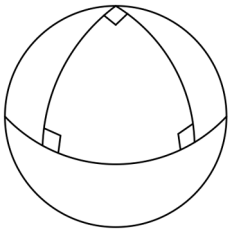
(E) $\frac{3}{8}$

[SOLN] There are $2^3 = 8$ possible sequences in three coin flips. Beginning with TT or HH just gets you back to Start (and you can't get from Start to End in one more turn). Only HTH gets from Start to End in exactly three flips. (The prob. of reaching End in exactly n turns ($n \geq 2$) is $\frac{F_{(n-1)}}{2^n}$ where F_n is the nth Fibonacci number.) □

11. The sum of the interior angles of a triangle on a sphere add up to more than 180° by an amount e called the *spherical excess*. The area of a spherical triangle is given by

$$A_{\triangle} = \frac{e}{720^\circ} A_{\text{sphere}}$$

How much of a sphere does a spherical triangle with three right angles cover?



[(A)] $\frac{1}{8}$
(B) $\frac{1}{4\pi}$
(C) $\frac{1}{4}$
(D) $\frac{3}{8}$
(E) $\frac{\pi}{4}$

[SOLN] Three right angles sum to 270° so $e = 90^\circ$. There are no similar (non-congruent) triangles on a sphere. □

12. A bookcase has 3 shelves. On the top shelf there are 10 German books, on the middle shelf 12 math books, and on the bottom shelf 8 science books. When six German books are removed from the bookcase, what fraction of the bookcase's remaining books are math books?

(A) $\frac{2}{5}$
(B) $\frac{1}{3}$
(C) $\frac{3}{5}$
[(D)] $\frac{1}{2}$
(E) $\frac{2}{3}$

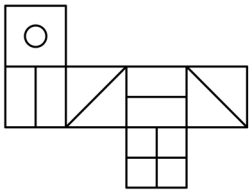
[SOLN] Removing 6 books leaves a total of 24, 12 of which are math books. □

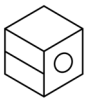
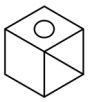
13. An item in a store is originally priced at \$4. A clerk marks the item up by 100%. After that, a second clerk marks the new price up by 200%. What is the final price after the two markups?

(A) \$12
(B) \$16
(C) \$18
(D) \$20
[(E)] \$24

[SOLN] \$4 marked up 100% becomes \$8. \$8 marked up 200% becomes \$24. □

14. Which box is formed from folding the figure on the right?



(A) 
(B) 
(C) 
(D) 
[(E)] 

[SOLN] B and C are eliminated because they have blank panels. A is eliminated because the diagonal line goes the wrong way. D is eliminated because the circle and four-square can't be adjacent. □

15. Evaluate: $4 + 4 \times 4 - 4 \div 4$

(A) 0

(B) 4

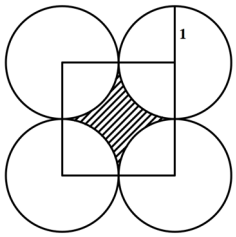
(C) 7

(D) 16

[(E)] 19

[SOLN] Order of operations says multiplication and division (left to right) first. $4 + 16 - 1$. Then go left to right. □

16. Find the area of the shaded region.



(A) 1

(B) $4/\pi$

[(C)] $4 - \pi$

(D) π

(E) 2π

[SOLN] Construct a square with corners at the centers of the four circles. The side of the square is 2 and the area is 4. The square contains four quarters of circles plus the shaded area, so $4 = 4(\frac{1}{4})(\pi(1)^2) + \text{shaded}$. □

17. What is the sum of all the one-digit prime numbers?

(A) 16

[(B)] $17 = 2 + 3 + 5 + 7$

(C) 18

(D) 27

(E) 45

[SOLN] 1 isn't prime. □

18. What is 0.3333 rounded to the nearest fifth?

(A) 0.1

(B) 0.2

(C) 0.3

[(D)] 0.4

(E) 0.5

[SOLN] The fifths between 0 and 1 (since $0 < 0.3333 < 1$) are 0.2, 0.4, 0.6, 0.8. Of those, 0.3333 is closest to 0.4. □

19. What is $6000\% + 600\% + 60\% + 6\%$?

(A) 6666

(B) 666.6

[(C)] 66.66

(D) 6.666

(E) 0.6666

[SOLN] $60\% = 0.6$; $600\% = 6$ □

20. The longest leg of a 30°-60°-90° triangle is 18 cm in length. What are the perimeter and area of the triangle?

(A) 26 cm, $54\,\text{cm}^2$

[(B)] $(18 + 18\sqrt{3})$ cm, $(54\sqrt{3})\,\text{cm}^2$

(C) 54 cm, $108\,\text{cm}^2$

(D) $(27 + 9\sqrt{3})$ cm, $(\frac{81}{2}\sqrt{3})\,\text{cm}^2$

(E) $(27\sqrt{2} + 18)$ cm, $(81\sqrt{2})\,\text{cm}^2$

[SOLN] The legs are 18 and $6\sqrt{3}$ so the hypotenuse is $12\sqrt{3}$. The sum is $18 + 18\sqrt{3}$ and the area is $\frac{1}{2}BH$. □