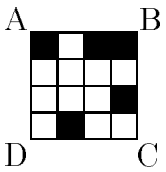
Snow College Jr. Mathematics Contest

key

April 1, 2008 **Junior division: grades 7–9** **Form: T**

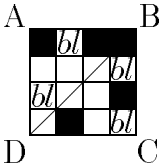
Bubble in the single best choice for each question you choose to answer.

1. What is the minimum number of small squares that must be darkened so the large square has BD as a line of symmetry?



(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

sorn For BD to be a line of symmetry the four small squares labeled *bl* must be colored black.



□

2. On a test, Barb beat Carli. Mark beat Bill. Kathy scored lower than Bill. Mike scored lower than Kathy but better than Barb. What was the order from first to last?

(A) Carli, Mark, Kathy, Mike, Barb, Bill
(B) Mark, Bill, Kathy, Carli, Mike, Barb
(C) Mike, Barb, Mark, Bill, Carli, Kathy
(D) Bill, Barb, Mark, Kathy, Mike, Carli
(E) Mark, Bill, Kathy, Mike, Barb, Carli

sorn "Barb beat Carli" eliminates A and B. "Mark beat Bill" eliminates D. "Kathy scored better than Barb" eliminates C. □

3. $\sqrt{100} = \sqrt{36} + \sqrt{?}$

(A) 2
(B) 4
(C) 16
(D) 64
(E) none of these

sorn $\sqrt{100} = 10$ and $\sqrt{36} = 6$ so $10 = 6 + x \Rightarrow x = 4$ so $? = 16$ □

4. If we were to write April 1, 2008 as a number (omitting punctuation) it would be 412008. What is the smallest prime number that is NOT a divisor of 412008?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

sorn 1 and 4 are not prime. 2 and 3 divide 412008, but 5 doesn't. □

5. How many different letters in the phrase "APRIL FOOL" do *not* have any reflection symmetries?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

sorn P, R, L, and F do not have any reflection symmetries. A, I, and O each have at least one reflection symmetry. □

6. Suppose that m people can do a task in d days. Assuming that they all work at the same rate, how many days will it take for $m+r$ people to do the task?

[(A)] $\frac{dm}{m+r}$

(B) $d - \frac{r}{m}$

(C) $\frac{d(m-r)}{m}$

(D) $\frac{dr}{m+r}$

(E) None of these

SOLN The number of man-days needed to complete the task is md; if we divide that by the number of people, $m+r$, then we'll have the number of days needed. □

7. What is the midpoint of the line segment between $(-4,3)$ and $(2,1)$?

[(A)] $(-1,2)$

(B) $(3,2)$

(C) $(2,3)$

(D) $(-0.5,1.5)$

(E) None of these

SOLN The midpoint of the line segment between (x_1,y_1) and (x_2,y_2) is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$. $(\frac{-4+2}{2}, \frac{3+1}{2}) = (-1,2)$ □

8. Geometry: if a ray begins inside of a simple polygon how many times might it intersect the polygon?

(A) 0

(B) 2

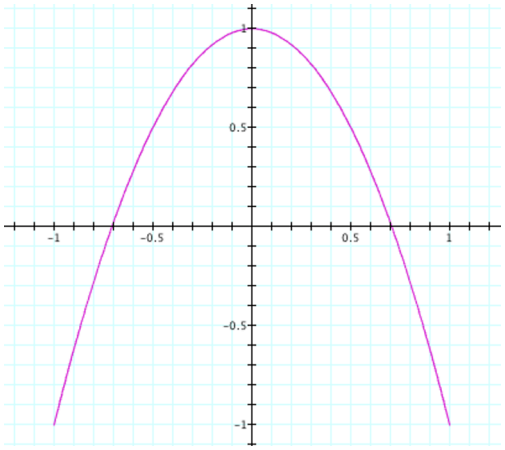
(C) 6

(D) 12

[(E)] 19

SOLN The number of intersections must be odd to get from inside to outside. □

9. Which set of parametric equations will produce the graph shown?



(A) $\begin{cases} x(t) = t \\ y(t) = t^2 \end{cases}$ $0 \le t \le 1$

(B) $\begin{cases} x(t) = t \\ y(t) = t^2 \end{cases}$ $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

(C) $\begin{cases} x(t) = t^2 \\ y(t) = t \end{cases}$ $-1 \le t \le 1$

(D) $\begin{cases} x(t) = t^2 \\ y(t) = t^2 \end{cases}$ $-1 \le t \le 1$

[(E)] $\begin{cases} x(t) = t \\ y(t) = -2t^2 + 1 \end{cases}$ $-1 \le t \le 1$

SOLN Do trial and error by plugging in the beginning, middle, and ending values for t into the equations for x and y to find a few ordered pairs (x,y). Or plug $x = t$ in $y(t) = -2t^2 + 1$ to get $y(x) = -2x^2 + 1$. This produces a parabola which opens down and peaks at $(0,1)$. □

10. If Tuesday, April 1, 2008 is the 92nd day of the year and July 4, 2008 is the 186th day of the year, what day of the week will July 4 fall on this year?

(A) Monday

(B) Tuesday

(C) Wednesday

(D) Thursday

[(E)] Friday

Soln $(186 - 92) \pmod 7 = 3$. Three days after Tuesday is Friday. □

11. Simplify. $\dfrac{5! \cdot 7!}{3! \cdot 6!}$

(A) 4!

(B) 80

[(C)] 140

(D) 35/18

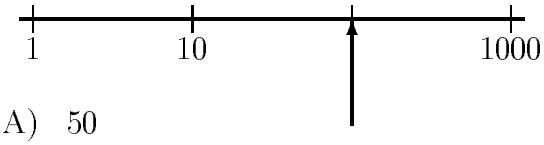
(E) 210

Soln

$$\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 4 \cdot 5 \cdot 7$$

□

12. The diagram shows the logarithmic scale on a measuring device. What is the approximate reading at the arrow?



(A) 50

[(B)] 100

(C) 200

(D) 500

(E) 750

Soln Since we have a logarithmic scale we are looking for a number whose $\log_{10}$ is 2, and that number is 100. □

13. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{2, 3, 5, 7\}$. What is $A \cap (B \cup C)$?

(A) $\{1, 2, 3, 4\}$

(B) $\{3, 4, 5\}$

[(C)] $\{2, 3, 4\}$

(D) $\{2, 3, 4, 5, 7\}$

(E) $\emptyset$

Soln $B \cup C = \{2, 3, 4, 5, 6, 7\}$. The intersection of that with A is $\{2, 3, 4\}$.□

14. Which statement best characterizes the points $(-1, -1)$, $(1, 2)$, and $(121, 182)$?

(A) They are not coplanar.

[(B)] They are collinear.

(C) They form an acute triangle.

(D) They form a right triangle.

(E) They form an obtuse triangle.

Soln The slope of the line segment between any pair of the points is the same $(\frac{3}{2})$, so the points must be collinear. □

15. Let $f(x, y) = x^2 - xy$. Then $f(x, x - y) =$

(A) x

(B) x^2y

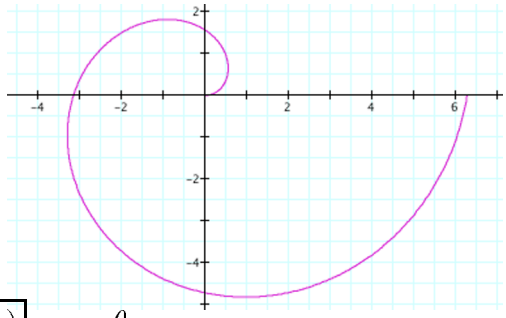
(C) $-y$

[(D)] xy

(E) $x^2(x - y)^2$

Soln $f(x, x - y) = x^2 - x(x - y) = x^2 - x^2 + xy$ □

16. In a polar plot the coordinates (r,θ) are used instead of (x,y), where r is the distance from the origin to the point and θ is the angle measured counterclockwise from the positive x-axis to the ray from the origin through the point. Which polar equation best describes the graph?



(A) $r = \theta$

(B) $r = \sin\theta$

(C) $r = \cos\theta$

(D) $r = 1$

(E) $\theta = 2\pi$

SOLN The point $(r,\theta) = (0,0)$ eliminates choices D and E. $r = \sin\theta$ and $r = \cos\theta$ are circles, so B and C are eliminated. As θ increases, so does r; this makes the spiral shown. □

17. What is $(2+3\mathrm{i})\cdot(2-3\mathrm{i})$?

(A) $(2-6\mathrm{i})$

(B) $(2+6\mathrm{i})$

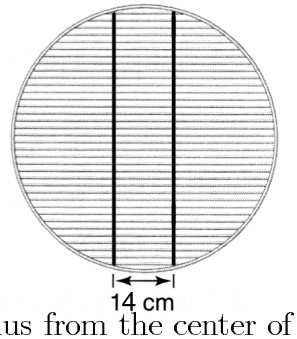
(C) $(4+9\mathrm{i})$

(D) -5

(E) 13

SOLN Use the FOIL method to get $4-(-9) = 13$. $(a+b)(a-b) = a^2 - b^2$ □

18. In the BBQ grill (diameter 50 cm) pictured, the two parallel support rods are equidistant from the center. What is the length of one of them?



(A) 46 cm

(B) 47 cm

(C) 48 cm

(D) 49 cm

(E) 50 cm

SOLN Draw a radius from the center of the grill to the end of a support rod. Use the Pythagorean theorem on the right triangle formed by the radius, half the support rod (call that x), and 7 cm. $25^2 - 7^2 = x^2$. Then $L = 2x$. □

19. The vector dot product is defined as

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

What is $(3,4,-5)\cdot(1,2,-1)$?

(A) $(6,-2,2)$

(B) $(31,42,-51)$

(C) 6

(D) 16

(E) $(4,6,-6)$

SOLN $(3)(1)+(4)(2)+(-5)(-1) = 16$□

20. It's Sophie's birthday! Sophie Germain, famous, self-taught, woman mathematician, was born April 1, 1776. A prime number p is called a *Sophie Germain prime* if $2p+1$ is also prime. Which of the following is not a Sophie Germain prime?

(A) 2

(B) 3

(C) 5

(D) 7

(E) 11

SOLN $2(7)+1 = 15$ which is not prime. □