Snow College Jr. Mathematics Contest

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.

2. Go ahead now and fill in the box at the top of your answer sheet. Print your name clearly, put your phone number in the "ID#" blank, spell out your school in the "class" blank, and put your year in school in the "sec" blank. Put your test version (Form A) in the "test no." blank. Also use a #2 (or HB or soft) pencil to bubble in your name on the left side of the answer sheet.

3. This is a one hour examination consisting of 20 multiple choice problems. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is   Score $= 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from -20 to 80.

4. In the event of a tie, the person with the largest number of the following five problems correct will be declared the winner: 2, 9, 10, 19, 20. Any further ties will be broken by a coin toss.

5. When the test begins, bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Use #2 (or soft) pencil. Completely erase any incorrect answers.

6. The sketches that accompany the problems are not necessarily drawn to scale.

7. No calculators are allowed.

8. Do not talk or disrupt other test takers during the exam. Cell phones must be OFF.

9. Please raise your hand if you need scratch paper; a proctor will assist you.

10. The proctors have been advised to answer no questions pertaining to the exam.

11. While we recommend you stay and recheck your answers if you have time, you may leave if you finish early (if you do, turn your answer sheet in and leave quietly). After the hour time limit is up the proctors will call for your answer sheets. Hand them in promptly.

After the test:

1. You may keep this test booklet.

2. If you RSVP'd to spend time with one of our science departments for lunch, please meet them in the science building; otherwise lunch may be purchased at the Snow College Cafeteria or downtown. In any event, you should plan to be back at the LDS Institute by 1:30 p.m. for the scores and presentation of the awards.

3. The top scorer from each grade 7–9 will be awarded a prize.

4. Thanks for coming. Your instructors will be happy to work the problems for you, and they will also be given copies of your answer sheets.

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Bubble in the single best answer to each question.

1. Find the area of the parallelogram.
   - (A) $35 \,\mathrm{m}^2$
   - (B) $70 \,\mathrm{m}^2$
   - (C) $50 \,\mathrm{m}^2$
   - (D) $34 \,\mathrm{m}^2$
   - (E) $149 \,\mathrm{m}^2$

   (10 m, 7 m, 7 m, 5 m, 10 m)
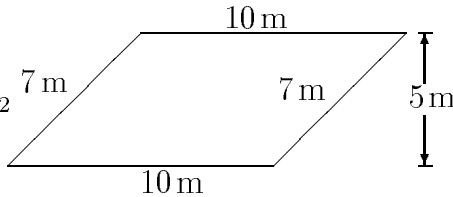
2. If you use the eight digits 1, 2, 3, 4, 5, 6, 7, and 9 each once and only once to form four 2-digit prime numbers, what will be the sum of the four prime numbers you created?
   - (A)   170
   - (B)   175
   - (C)   180
   - (D)   185
   - (E)   190

   There are multiple possiblities but we do know that no 2-digit even numbers are prime; nor are numbers ending in 5. Therefore the digits 2, 4, 6, and 5 must be in the tens place. That leaves 1, 3, 7, and 9 in the ones place.

3. Write 0.00750×10^8 in scientific notation.
   - (A) 7.5×10^4
   - (B) 7.50×10^4
   - (C) 7.5×10^5
   - (D) 7.50×10^5
   - (E) 7.5×10^6

   The trailing zero is significant.

4. The five square numerical puzzle requires the vertical column to be filled with the digits (in the correct order) from a positive 3-digit integer power of 5, and the horizontal row to be filled with the digits (in the correct order) from a positive 3-digit integer power of 2. What will be the digit in the rightmost square?
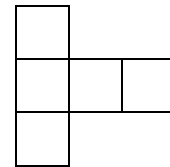   - (A)   6
   - (B)   7
   - (C)   8
   - (D)   9
   - (E)   0

   The only 3-digit integer powers of 5 are 125 and 625; in either case the middle digit is 2. The only 3-digit power of 2 that starts with 2 is 256.

5. If three distinct counting numbers have a sum of 10 and a product of 20, then what is their median (*i.e.*, the middle number)?
   - (A)   2
   - (B)   3
   - (C)   4
   - (D)   5
   - (E)   not enough information

   The numbers are 1, 4, and 5.

6. A superball rebounds half the height from which it is dropped. If you drop the ball straight down from a starting height of 32 feet, what is the total distance (up and down) the ball will have traveled when it strikes the ground for the fourth time?

(A) 64 ft

(B) 74 ft

(C) 88 ft

(D) 92 ft

(E) 120 ft

$$32 + 2(16) + 2(8) + 2(4) = 88$$

7. Sue had walked halfway from home to school when she realized she was late. She ran the rest of the way to school. She ran 3 times as fast as she walked. Sue took 6 minutes to walk halfway to school. How many minutes did it take Sue to get from home to school?

(A) 7

(B) 7.3

(C) 7.7

(D) 8

(E) 8.3

Covering the same distance three times as fast takes one-third the time. So Sue ran for 2 minutes. Her total time was $6 + 2 = 8$ minutes.

8. The area of the rectangle $ABCD$ is $48\,\text{in}^2$ and the length of its diagonal is 10 in. What is the perimeter of the rectangle?

(A) 8 in

(B) 12 in

(C) 18 in

(D) 20 in

(E) 28 in
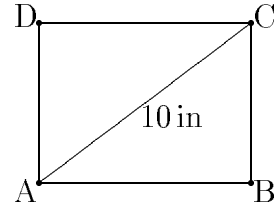
Let x be the length of side AB and y be the length of side BC. Then $xy = 48$ and $x^2 + y^2 = 100$. The factors of 48 suggest $x = 8$ and $y = 6$ and those choices also give the correct proportions for a 3-4-5 right triangle.

9. If the length of a rectangle were increased by 20% and the width by 25%, by what percent would the area increase?

(A) 27.5%

(B) 35%

(C) 45%

(D) 50%

(E) 65%

The area of a rectangle is $l \times w$. If l is increased by 20% and the width by 25%, the new area would be $(1.20l \times 1.25w) = 1.50 \times l \times w$.

10. If $x + 2y = 84 = 2x + y$, what is the value of $x + y$?

   [(A)]   56

   (B)   62

   (C)   66

   (D)   74

   (E)   84

   > Since $x + 2y = 84$ and $2x + y = 84$, then add the two equations together to obtain $3x + 3y = 168$. Divide by 3 to get $x + y = 56$.
   >
   > Or, since the two equations are identical when x is replaced with y and vice versa, then $x = y$. So $3x = 84 \Rightarrow x = 28$.

11. How many prime numbers are there between 10 and 30?

   (A)   fewer than four

   (B)   four

   (C)   five

   [(D)]   six

   (E)   more than six

   > 11, 13, 17, 19, 23, 29

12. As shown in the figure, a triangular corner with side lengths $DB = EB = 1$ is cut from equilateral triangle ABC of side length 3. What is the perimeter of the remaining quadrilateral $ADEC$?

   (A)   6

   (B) $6\frac{1}{2}$

   (C)   7

   (D) $7\frac{1}{2}$

   [(E)]   8
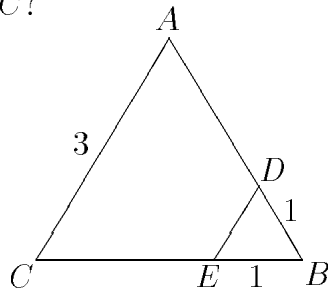
   > $AD = CE = 2$ and $ED = 1$ so the total perimeter is $3 + 2 + 2 + 1$.

13. Here's a puzzler from NPR's Car Talk. Tommy says, "Bugsy owns 50 cars, maybe more." Berman says, "No, no, no, he owns fewer than 50 cars, I know that for sure." Then Cronin says, "I know one thing, he owns at least one car." Only one of these statements is true. How many cars does Bugsy really own?

   (A)   49

   (B)   50

   (C)   51

   (D)   not enough information

   [(E)]   none of the above

   > For only one of the statements to be true it must be the second one. So the third statement is false. Bugsy owns zero cars.

14. On a musical instrument with 4 valves, how many different fingerings are possible?

   (A)   8

   (B)   10

   (C)   12

   [(D)]   16

   (E)   20

   > Each valve can be up or down (two positions), so the answer is $2^4 = 16$.

15. When the fog lifted, 400 ghouls were spied skulking near the outskirts. If 240 ghouls were not spied, what fraction of the ghouls were not spied?

   (A) $\frac{3}{5}$

   [(B)] $\frac{3}{8}$

   (C) $\frac{5}{8}$

   (D) $\frac{5}{3}$

   (E)   none of the above

   > The total number of ghouls is $240 + 400 = 640$; so the fraction not spied is $\frac{240}{640}$.

16. Newton's law of gravitation gives the strength of the gravitational force between objects of mass m_1 and m_2 as

$$F = G\frac{m_1 m_2}{d^2}$$

where G is the universal gravitational constant and d is the distance between the objects. If the force between two objects is 16 newtons when they are 1 m apart, what is the force when they are 4 m apart?

(A) 2 newtons

(B) 4 newtons

(C) 8 newtons

(D) 64 newtons

(E) none of the above

    Increasing the distance by a factor of 4 decreases the force by a factor or 4^2 so the new force is 1 newton.

17. Add (write with all positive exponents).

$$a^{-2}xy^{-1} + az^{-1}$$

(A) $\frac{x^2 + a^2 y}{a^3 yz}$

(B) $\frac{xy + z^3 y}{x^2 az}$

(C) $\frac{yz + a^3 z}{xy^2 z}$

(D) $\frac{a^3 z + ax}{a^2 xy}$

(E) $\frac{xz + a^3 y}{a^2 yz}$

    $\frac{x}{a^2 y} + \frac{a}{z} = \frac{xz}{a^2 yz} + \frac{a^3 y}{a^2 yz}$

18. What is the sum of the exponents in the prime factorization of 120?

(A) 3

(B) 4

(C) 5

(D) 6

(E) 7

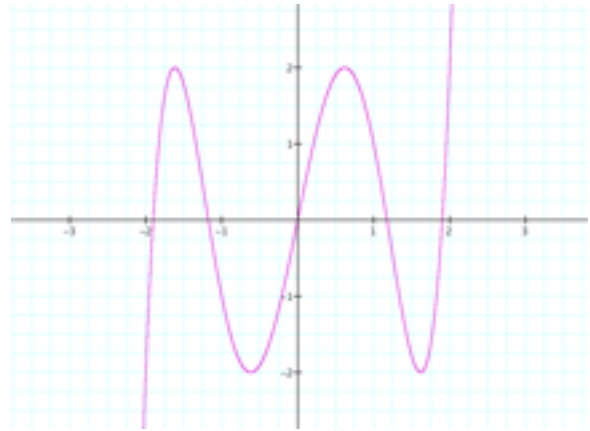   The prime factorization is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 2^3 \cdot 3^1 \cdot 5^1$.

19. What is the sum of the solutions of $x^2 - 5x + 6 = 0$?

(A) -5

(B) -6

(C) 5

(D) 6

(E) none of the above

    The factorization is $(x-3)(x-2)$.

20. Which function does the graph represent?

(A) $y = 2\cos 2x$

(B) $y = 4x^2 - 5x + 3$

(C) $y = 2x^3 + 3x^2 - x$

(D) $y = x^4 + 2x^2 + 1$

(E) $y = x^5 - 5x^3 + 5x$

    Because it goes up to the right and down to the left we think it is a polynomial of odd degree. Because it has 4 local extreme values, the polynomial must be at least degree 5. We also know the degree is at least 5 because it crosses the x-axis five times. All the other choices are also easily eliminated for various other reasons.