Snow College Mathematics Contest

key

April 5, 2016 **Senior Division: Grades 10-12** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Using a scale, you find that four t**w**eezers and a **b**ar of soap weigh the same as three **c**ombs; a bar of soap weighs the same as two **t**oothbrushes and a comb; and six tweezers weigh the same as one toothbrush and one comb. How many combs are needed to equal the weight of one bar of soap?

   (A) 2 ✓
   (B) 4
   (C) 6
   (D) 8
   (E) 10

   SOLN Let w be tweezers, t toothbrushes. (1) $4w + b = 3c$, (2) $b = 2t + c$, and (3) $6w = t + c$. Eliminating w from the system made up of equations (1) and (3) gives $0 = 7c - 3b - 2t$. Eliminating t from this new equation and equation (2) gives $b = 2c$. □

2. On one moving sidewalk in an airport it takes 2 min to stand the entire ride. Jamie can walk the same distance in 90 s. How long would it take Jamie to walk from start to end on the moving sidewalk?

   (A) 51 s ✓
   (B) 47 s
   (C) 65 s
   (D) 58 s
   (E) 55 s

   SOLN t = time, d = distance, r = rate subscripts: walk **on**, walk **off**, **st**and $t_{\text{on}} = \frac{d}{r_{\text{on}}} = \frac{d}{r_{\text{st}}+r_{\text{off}}} = \frac{d}{\frac{d}{t_{\text{st}}}+\frac{d}{t_{\text{off}}}} = \frac{t_{\text{st}}t_{\text{off}}}{t_{\text{st}}+t_{\text{off}}} = \frac{(2\,\text{min})\left(\frac{3}{2}\,\text{min}\right)}{(2\,\text{min})+\left(\frac{3}{2}\,\text{min}\right)} = \frac{6}{7}\,\text{min}$ □

3. A fountain spouts water from two eyes, a mouth, and a foot. The right eye would fill a specific jar in two days, the left eye in three days, the mouth in six hours, and the foot in four days. To the nearest hour, how long will it take all four of these spouts together to fill the jar?

   (A) 2
   (B) 3
   (C) 4
   (D) 5 ✓
   (E) 6

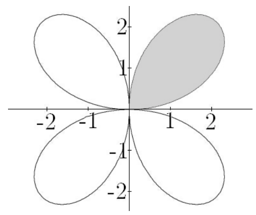
   SOLN $\left(\frac{1}{2} + \frac{1}{3} + 4 + \frac{1}{4}\right) t = 1$, where t is the time required to fill the jar using all spouts. Solving gives $t = \frac{12}{61}\,\text{d} \approx \frac{12}{60}\,\text{d} = \frac{1}{5}\,\text{d} \approx \frac{5}{25}\,\text{d} \approx \frac{5}{24}\,\text{d} \approx 5\,\text{h}$. Alt soln: LCM = 12 d. In 12 d the right eye fills 6 jars, left eye 4 jars, mouth 48 jars, and foot 3 jars. This is 61 jars in 12 days or 12/61 days per jar. Adapted from a Greek Anthology by Metrodorus circa 500 A.D. □

4. What is the slope of the line tangent to $y = x^3$ at $(0, 0)$?

   (A) $\frac{\pi}{4}$
   (B) $\frac{7}{4}$
   (C) $-\frac{7}{4}$
   (D) 0 ✓
   (E) -1

   SOLN Evaluate the derivative at $x = 0$. $y' = 3x^2$ At $x = 0$, $3x^2 = 0$. Note, this is a point of inflection where the concavity changes. □

5. Find the area inside one leaf of the curve $r = 3\sin(2\theta)$.



$\boxed{\text{(A)}}$ $\frac{9\pi}{8}$

(B) $\frac{3\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) $\frac{9\pi}{4}$

(E) π

$\boxed{\mathcal{SOLN}}$ $\frac{1}{2}\int_\alpha^\beta r^2 \mathrm{d}\theta = \frac{1}{2}\int_0^{\pi/2} (3\sin(2\theta))^2 \mathrm{d}\theta = \frac{9}{2}\int_0^{\pi/2} \frac{1-\cos(4\theta)}{2} \mathrm{d}\theta = \frac{9}{4}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2} = \frac{9}{4} \cdot \frac{\pi}{2}$ $\square$

6. Consider the equation $x^2 + bx + 2 = 0$. A single, fair, 6-sided die is rolled to determine the value of the middle coefficient b which becomes the number of pips on the upper face of the die. What is the probability that the equation will have real unequal roots?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

$\boxed{\text{(D)}}$ $\frac{2}{3}$

(E) $\frac{5}{6}$

$\boxed{\mathcal{SOLN}}$ The discriminant of the quadratic equation is $\sqrt{b^2 - 8}$ and is a real non-zero number for $b = 3, 4, 5, 6$. $\square$

7. The base three representation of x is $12\,11\,22\,11\,12\,22\,11\,11\,22\,22$. Find the left digit of the base nine representation of x.

(A) 1

(B) 2

(C) 3

(D) 4

$\boxed{\text{(E)}}$ 5

$\boxed{\mathcal{SOLN}}$ Since $3^2 = 9$, group by two digits starting from right. Left group is 12. $12_{\text{three}} = 5$ $\quad x_{\text{nine}} = 5484584488$ $\square$

8. What is the value of the expression?

$$\sqrt{16 + \sqrt{16 + \sqrt{16 + \dots}}}$$

(A) $2\sqrt{2}$

(B) 4

(C) 4.52

(D) $-\frac{1}{2} + \frac{\sqrt{65}}{2}$

$\boxed{\text{(E)}}$ $\frac{1}{2} + \frac{\sqrt{65}}{2}$

$\boxed{\mathcal{SOLN}}$ Let $\sqrt{16 + \sqrt{16 + \sqrt{16 + \dots}}} = x$ $\sqrt{16 + x} = x \implies 16 + x = x^2$. Use the quadratic formula and take the positive value: $x = \frac{1+\sqrt{65}}{2}$ $\square$

9. If $\log_4(\log_4(\log_4(\log_4(x)))) = 0$, what is x?

(A) 1

(B) 256^3

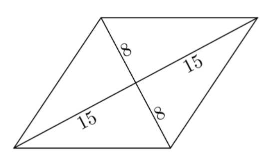
(C) 4^{16}

$\boxed{\text{(D)}}$ 2^{512}

(E) 256^4

$\boxed{\mathcal{SOLN}}$ Exponentiate 4 times: $4^{4^{4^{4^0}}} = 4^{4^{4^1}} = 4^{4^4} = 4^{256} = (2^2)^{256}$ $\square$

10. Find the area and perimeter of the rhombus.



(A) $A = 480, P = 88$

(B) $A = 289, P = 68$

$\boxed{\text{(C)}}$ $A = 240, P = 68$

(D) $A = 225, P = 60$

(E) $A = 120, P = 40$

$\boxed{\mathcal{SOLN}}$ Area of a rhombus is half the product of the diagonals. $A = \frac{d_1 d_2}{2} = 16 \cdot 30/2 = 240$. The diagonals are perpendicular so we can use the Pythagorean thm: the sides are 17. $P = 4s = 4(17) = 68$ $\square$

11. If $a + b = 3$ and $a^2 + b^2 = 89$, then what is $a^3 + b^3$?

(A) 307

(B) 347

$\boxed{\text{(C)}}$ 387

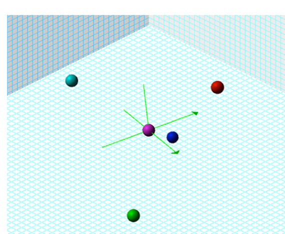
(D) 507

(E) Not possible to determine

$\boxed{\mathcal{SOLN}}$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ so $a^3 + b^3 = 3(89 - ab)$. Note $9 = (a+b)^2 = (a^2 + 2ab + b^2)$, so $ab = -40$. $\square$

12. How many ordered triples (x, y, z) of real numbers satisfy the conditions?

$$xy = z, \quad xz = y, \quad yz = x$$



(A) 1

(B) 2

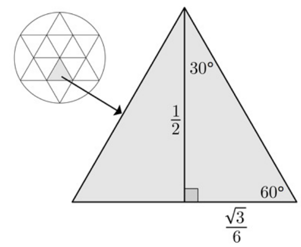
(C) 3

(D) 4

$\boxed{\text{(E)}}$ 5

$\boxed{\mathcal{SOLN}}$ If one of x, y, z is 0 then all three are, yielding triple $(0, 0, 0)$. If none are 0 then each of x, y, z must be ± 1. $(1, 1, 1)$ works. If any is negative then exactly two of them are: $(1, -1, -1)$, $(-1, 1, -1)$, $(-1, -1, 1)$. $\square$

13. Two equilateral triangles are arranged into a perfectly symmetrical "Star of David" and circumscribed by a circle of radius 1. What is the area of the regular hexagon formed by the intersection of the two triangles?



(A) 1

$\boxed{\text{(B)}}$ $\frac{\sqrt{3}}{2}$

(C) $\frac{\pi}{2}$

(D) $\sqrt{3}$

(E) π

$\boxed{\mathcal{SOLN}}$ The star can be divided up into 12 congruent equilateral triangles as shown. The radius of the circle is the sum of the heights of two of them. Drawing a radius of the circle to one of the stars corners, then we see that each of these triangles has a height of $\frac{1}{2}$. Using rules of 30-60-90 triangles we find the area of each triangle is $\sqrt{3}/12$ so the area of the hexagon is $6 \cdot \sqrt{3}/12 = \sqrt{3}/2$. $\square$

14. $\frac{68}{77}$ can be expressed as the sum of two positive proper fractions whose denominators are 7 and 11: $\frac{a}{7} + \frac{b}{11}$. What is $a + b$?

(A) 6

$\boxed{\text{(B)}}$ 8

(C) 11

(D) 17

(E) $8\sqrt{5}$

$\boxed{\mathcal{SOLN}}$ $11a + 7b = 68$ $\qquad \frac{68}{77} = \frac{3}{7} + \frac{5}{11}$ $\square$

15. If $\sin x = 3\cos x$, what is $\sin x \cos x$?

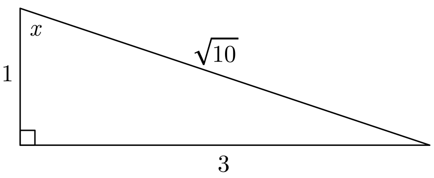
(A) $\frac{1}{3}$

$\boxed{\text{(B)}}$ $\frac{3}{10}$

(C) $\frac{2}{3}$

(D) $\frac{5}{6}$

(E) $\frac{3}{2}$



SOLN Divide by $\cos x$. $\tan x = 3$.
See figure. $\sin x \cos x = \left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right)$
Soln 2: Multiplying the equation by $\cos x$ gives $\sin x \cos x = 3\cos^2 x$, whereas multiplying by $\sin x$ gives $\sin^2 x = 3\cos x \sin x$. Add the first result to three times the second to obtain $10 \sin x \cos x = 3$.
Soln 3: Square both: $\sin^2 x = 9\cos^2 x$
$1 - \cos^2 x = 9\cos^2 x \Rightarrow \frac{1}{10} = \cos^2 x$ □

16. One hundred tickets numbered 1 through 100 are in a bowl. The tickets are drawn one at a time without replacement. How many tickets must be drawn to be sure that one of the selected numbers is exactly twice as large as another selected number?

(A) 50

(B) 63

$\boxed{\text{(C)}}$ 68

(D) 75

(E) 84

SOLN We want the cardinality of the largest set without any doubles of another number in the set. Start at 100 and draw all the way down to 51. Now don't draw 26–50 since each of them have a double in the drawn set. Draw 13–25 but not 7–12. Draw 4–6 but not 2, 3. Draw 1. You've drawn $50 + 13 + 3 + 1 = 67$ tickets. The next one drawn must have a double in the drawn set. □

17. In 6 years Bill's age will be a perfect square. Six years ago Bill's age was the square root of that perfect square. How old is Bill?

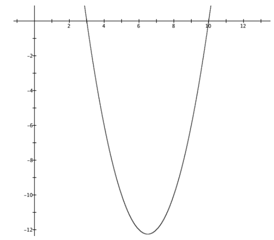
(A) 6

(B) 8

$\boxed{\text{(C)}}$ 10

(D) 12

(E) 14



SOLN Solve $a+6 = (a-6)^2$ where $a - 6 \geqslant 0$.
$a^2 - 13a + 30 = 0$ □

18. Antoine, Benoît, Claude, Didier, Étienne, and François go to the cinema. They want to sit in a single row of six seats, but A, B, and C are mutual enemies and refuse to sit next to either of the other two. How many different arrangements are possible?

(A) 169

(B) 121

(C) 136

$\boxed{\text{(D)}}$ 144

(E) 125

SOLN A, B, and C must take seats $(1, 3, 5)$, $(1, 3, 6)$, $(1, 4, 6)$, or $(2, 4, 6)$. D, E, and F take the other three seats in any order. Within each grouping there are 3! ways to arrange. Total is $4 \cdot 3! \cdot 3! = 144$ □

19. A and B want to know when C's birthday is. C gives them a list of 10 possible dates.

May 15	May 16	May 19
June 17	June 18	
July 14	July 16	
August 14	August 15	August 17

C then tells A and B separately the month and the day of her birthday respectively.

A: I don't know when C's birthday is, but I know that B does not know either.
B: At first I didn't know when C's birthday is, but I know now.
A: Then I also know when C's birthday is.

So when is C's birthday?

(A) May 16

(B) May 19

(C) June 17

(D) July 14

$\boxed{\text{(E)}}$ July 16

$\boxed{\text{SOLN}}$ A knows B can't know by date alone so it isn't 18 or 19. A can only know this if he knows the month isn't May or June. For B to know the date with certainty, the date can't be 14. For A to know, the month must be July. □

20. A merchant buys some kiwis at 3 for 10¢ and an equal number at 5 for 20¢. What should the selling price be to "break even"?

(A) 8 for 30¢

$\boxed{\text{(B)}}$ 3 for 11¢

(C) 5 for 18¢

(D) 11 for 40¢

(E) 13 for 50¢

$\boxed{\text{SOLN}}$ Let n be the number bought at each price and x be the selling price.
$2nx = \frac{10}{3}n + \frac{20}{5}n \implies x = 11/3$ □

21. A golden triangle is an isosceles triangle in which the ratio of the duplicated side to the distinct side is $\phi = (1+\sqrt{5})/2$. Unique to golden triangles is that the three angles have a 2:2:1 proportion. How many non-overlapping golden triangles can be arranged around a common vertex?

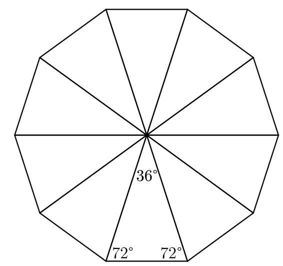
$\boxed{\text{(A)}}$ 10

(B) 12

(C) 15

(D) 18

(E) 20



$\boxed{\text{SOLN}}$ Call the vertex angle (in degrees) n. Since the proportion is 2:2:1 we have $2n + 2n + n = 180 \implies n = 36$. Therefore $360/36 = 10$ of them with a common vertex make a regular decagon. □

22. The strength of the electric force between two particles with charge q_1 and q_2 a distance r apart is

$$F = k\frac{|q_1|\,|q_2|}{r^2}$$

If the distance between two charged particles is trebled (tripled), what is the ratio of the new force to the original force?

(A) 9:1

(B) 3:1

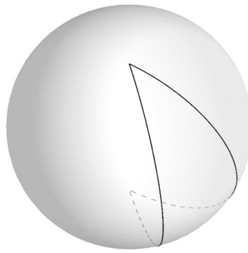
(C) 1:1

(D) 1:3

$\boxed{\text{(E)}}$ 1:9

$\boxed{\text{SOLN}}$ Because it is an inverse square law, trebling the distance decreases the force by a factor of 9. □

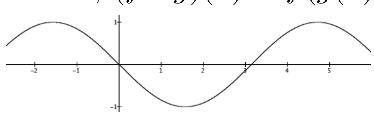
23. The simplest polygon in a plane is a triangle (whose angles sum to 180°). On a sphere the simplest polygon is a *biangle* (or *lune*). What is the range for the sum S of the angles in a biangle confined to one hemisphere?

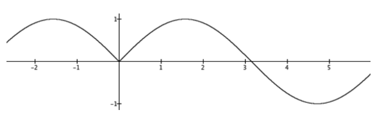


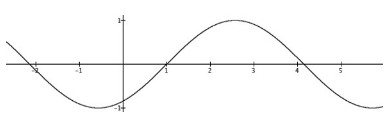
$\boxed{\text{(A)}}$ $0^\circ < S < 360^\circ$
(B) $45^\circ < S < 360^\circ$
(C) $45^\circ < S < 180^\circ$
(D) $0^\circ < S < 180^\circ$
(E) $90^\circ < S < 180^\circ$

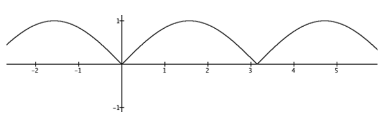
$\boxed{\text{SOLN}}$ On a sphere straight lines are great circles. Two great circles intersect antipodally. Each of the 2 angles (which must be equal) can range between 0° and 180° (exclusive, to eliminate degenerate cases). □

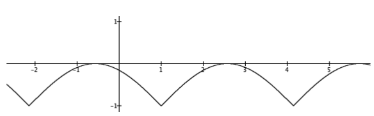
24. Given $f(x) = |x-1| - 1$ and $g(x) = \sin x$, which is a graph of the composition of the two functions, $(f \circ g)(x) = f(g(x))$?

$\boxed{\text{(A)}}$ 

(B) 

(C) 

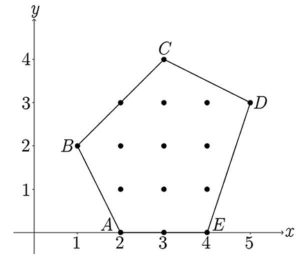
(D) 

(E) 

$\boxed{\text{SOLN}}$ $(f \circ g)(x) = |\sin x - 1| - 1$

Graphs made with Graphing Calculator. □

25. Let $A = (2,0)$, $B = (1,2)$, $C = (3,4)$, $D = (5,3)$, and $E = (4,0)$. Find the area, in square units, of pentagon $ABCDE$.



$\boxed{\text{(A)}}$ 10.5
(B) 12
(C) 9
(D) 10
(E) 11.5

$\boxed{\text{SOLN}}$ Long way: break up pentagon into triangles and rectangles and find the area of each piece. Short way: use Pick's theorem: the area of a simple polygon whose vertices are grid points is $A = I + B/2 - 1$ where I is the number of interior grid points and B is the number of boundary grid points. $A = 8 + 7/2 - 1 = 21/2$ □

26. Flip a fair coin 10 times. What is the probability that at least 3 heads appear?

$\boxed{\text{(A)}}$ $\frac{968}{1024}$
(B) $\frac{992}{1024}$
(C) $\frac{946}{1024}$
(D) $\frac{1013}{1024}$
(E) $\frac{923}{1024}$

$\boxed{\text{SOLN}}$ $2^{10} = 1024$. There is 1 way to get 0 heads, $\binom{10}{1} = 10$ ways to get exactly 1 head, and $\binom{10}{2} = 45$ ways to get exactly 2 heads. $\frac{1024-1-10-45}{1024}$ □

27. On what interval(s) is the graph of $h(x) = x^3 - x^2 + 5$ concave up?

(A) $(-\infty, 0)$ and $(\frac{2}{3}, \infty)$
(B) $(-\infty, \frac{1}{3})$
(C) $(3, \infty)$
$\boxed{\text{(D)}}$ $(\frac{1}{3}, \infty)$
(E) $(\frac{1}{3}, \frac{2}{3})$

$\boxed{\text{SOLN}}$ $h''(x) = 0 \implies x = \frac{1}{3}$. Check sign of $h''(x)$ above and below. □

28. Given: a, b, c, d are non-zero real numbers and $a + b\mathrm{i}$ and c are roots of the polynomial $x^4 + dx^3 + dx + 1$ (a palindromic polynomial because of the symmetry of the coefficients). What is the sum of all real roots?

(A) $2a$

$\boxed{\text{(B)}}$ $\frac{c^2+1}{c}$

(C) $a + c$

(D) $2a + 2c$

(E) $2c$

SOLN There are two complex roots, and two real roots. If c is a root, then plug in $1/c$ and multiply by c^4; the result is the original polynomial with c plugged in, which is 0 (by hypothesis). So the sum of the real roots is $c + \frac{1}{c}$. □

29. A *fixed point* of a set S_n of permutations (or reorderings) of n objects is a number in the list $\{1, 2, \ldots, n\}$ that the permutation leaves alone. For example, $1 \mapsto 2$, $2 \mapsto 1$, $3 \mapsto 3$ is a reordering of $\{1, 2, 3\}$ to the permutation $\{2, 1, 3\}$ which leaves 3 alone. For $n = 4$ how large is the largest subset of permutations in S_4 such that every permutation in that subset shares the same fixed point?

(A) 1

(B) 3

(C) 4

$\boxed{\text{(D)}}$ 6

(E) 12

SOLN The set of all permutations in S_4 that fix one particular point is isomorphic to S_3 which has size $3 \cdot 2 \cdot 1$. That is, if you "fix" one point then there are 3 places for a remaining value, 2 for the next, and 1 for the last. □

30. What is the smallest integer that can be written in two different ways (different numbers) as the sum of two positive cubes?

(A) 341

(B) 728

(C) 1339

$\boxed{\text{(D)}}$ 1729

(E) 4104

SOLN $1729 = 1^3 + 12^3 = 9^3 + 10^3$. To see that this is the smallest, consider all 66 possible sums of combinations of pairs of the first 12 positive cubes (1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728). Story at sumofcubes.weebly.com. □

31. What x-values satisfy $\sqrt{x^2 + y^2} = 5 - x$?

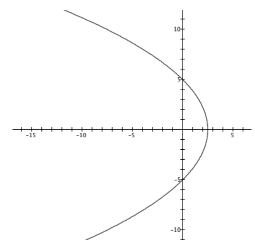
(A) $x \in (-\infty, \infty)$

$\boxed{\text{(B)}}$ $x \in \left(-\infty, \frac{5}{2}\right]$

(C) $x \in [-5, 5]$

(D) $x = 0$

(E) $x \in \left[\frac{5}{2}, 5\right]$



SOLN Graph the equation: $r = 5/(1 + \cos\theta)$ in polar coordinates. Parabola opening to the left with vertex at $x = \frac{5}{2}$. □

32. What is the smallest positive integer n such that $1 + 2 + 3 + \ldots + n > 5000$?

(A) 90

(B) 99

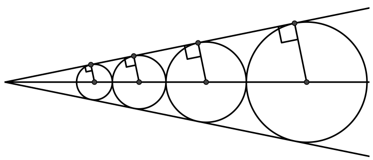
$\boxed{\text{(C)}}$ 100

(D) 101

(E) 110

SOLN The sum of the first n positive integers is $\frac{n(n+1)}{2}$. The solution is the first positive n for which $n(n + 1) > 10000$. When is the product of consecutive positive integers first larger than $10000 = 100^2$? Clearly, $99 \cdot 100 < 100 \cdot 100 < 100 \cdot 101$; therefore $n = 100$. □

33. Four circles are placed in an angle so that they all just fit as shown. If the radius of the smallest is 16 and the radius of the largest is 54, what are the radii of the two circles in between?



(A) $22\frac{2}{3}, 35\frac{1}{3}$

$\boxed{\text{(B)}}$ $24, 36$

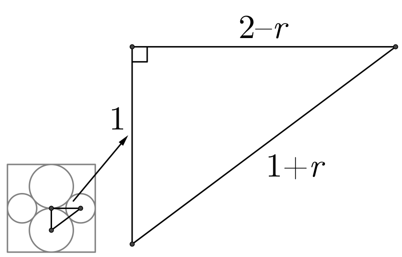
(C) $27, 42$

(D) $28\frac{2}{3}, 39\frac{1}{3}$

(E) $30, 45$

$\boxed{\mathcal{SOLN}}$ By angle-angle similarity, the ratio of radii of any adjacent pair must be the same. $\frac{16}{r_1} = \frac{r_1}{r_2} = \frac{r_2}{54}$. Two independent equations with two unknowns. For example, $r_1^2 = 16r_2$ and $r_2^2 = 54r_1 \implies r_2^3 = 16 \cdot 54^2 = 2^6 \cdot 3^6$ □

34. In the figure all four circles are tangent to the square, the two large circles have the same radius and are tangent to each other, and the two small circles have the same radius and are tangent to each of the large circles. If the radius of the large circles is 1, what is the radius r of the small circles?



(A) $\frac{1}{2}$

$\boxed{\text{(B)}}$ $\frac{2}{3}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\frac{3}{4}$

(E) $\frac{\sqrt{3}}{2}$

$\boxed{\mathcal{SOLN}}$ Construct a right triangle with legs 1 and $2 - r$ and hypotenuse $1 + r$. Pythagorean Thm gives $r = \frac{2}{3}$.

From a child's Sangaku puzzle in 1847. □

35. Compute the value of the limit. $\lim_{x \to 0} \dfrac{\tan x}{2x}$

(A) 0

$\boxed{\text{(B)}}$ $\frac{1}{2}$

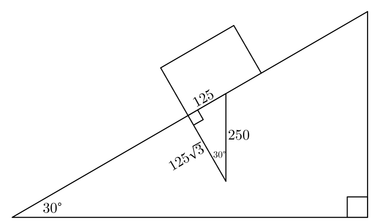
(C) 1

(D) 2

(E) The limit does not exist

$\boxed{\mathcal{SOLN}}$ $\dfrac{\tan x}{2x} = \dfrac{\sin x}{x} \dfrac{1}{2\cos x}$. The limit of the first fraction as $x \to 0$ is 1 and the limit of the second is $\frac{1}{2}$. □

36. Newton's laws tell us that for every force exerted by an object, there is an equal and opposite force exerted back on that object. If a box is held in place on a 30° inclined plane by a static friction force of 125 newtons, what is the magnitude of the force exerted by the plane on the box?



(A) $0\,\text{N}$

(B) $62.5\,\text{N}$

(C) $\frac{125\sqrt{3}}{3}\,\text{N}$

(D) $125\,\text{N}$

$\boxed{\text{(E)}}$ $125\sqrt{3}\,\text{N}$

$\boxed{\mathcal{SOLN}}$ Draw a 30-60-90 right triangle with the friction force as the short leg and the normal force as the longer leg. $\tan 60^\circ = |\vec{F}_\text{n}|/125\,\text{N}$ □

37. Two objects are topologically equivalent if we can stretch, shrink, bend, or twist one, without cutting or gluing, and deform it into the other. Which is equivalent to **F**?

(A) **G**

(B) **L**

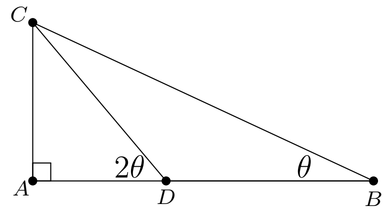
(C) **W**

(D) **X**

$\boxed{\text{(E)}}$ **Y**

$\boxed{\mathcal{SOLN}}$ The branches of **Y** can be transformed into the branches of **F**. Each has one point from which you can go in three directions. □

38. In triangle ABC, given $m\angle ABC = \theta$ and $m\angle ADC = 2\theta$, which formula correctly gives the length of side AC in terms of the length of side BD and the angle θ?



(A) $AC = BD \tan 3\theta$

(B) $AC = BD \cos \theta$

(C) $AC = BD \cos 2\theta$

(D) $AC = BD \sin \theta$

$\boxed{\text{(E)}}$ $AC = BD \sin 2\theta$

$\boxed{\mathcal{SOLN}}$ $m\angle BDC = 180° - 2\theta$ so $m\angle BCD = 180° - \theta - (180° - 2\theta) = \theta$ so $CD = BD$. Since $\sin 2\theta = \frac{AC}{CD}$, $AC = CD \sin 2\theta = BD \sin 2\theta$. □

39. Evaluate. $(1 + \mathrm{i})^2$

(A) $-2 - \mathrm{i}$

(B) $-1 + \mathrm{i}$

$\boxed{\text{(C)}}$ $2\mathrm{i}$

(D) $1 - 2\mathrm{i}$

(E) $2 + 2\mathrm{i}$

$\boxed{\mathcal{SOLN}}$ $1 \cdot 1 + 1 \cdot \mathrm{i} + \mathrm{i} \cdot 1 + \mathrm{i} \cdot \mathrm{i} = 1 + 2\mathrm{i} - 1$
Soln 2: If $x, y \in \mathbb{C}$ and $z = xy$, then $\angle z = \angle x + \angle y$ and $|z| = |x|\,|y|$. $\angle(1 + \mathrm{i}) = 45°$ and $|1 + \mathrm{i}| = \sqrt{2}$ □

40. What is the output of the following BASIC computer program?

```
10 for i = 1 to 5
20 print i^i/i
30 next i
```

$\boxed{\text{(A)}}$ `1 2 9 64 625`

(B) `1 2 3 4 5`

(C) `1 4 27 256 3125`

(D) `1 4 9 16 25`

(E) `1 32 243 1024 3125`

$\boxed{\mathcal{SOLN}}$ Order of operations: $(i^i)/i = i^{(i-1)}$ □