Snow College Mathematics Contest

April 5, 2016 **Senior Division: Grades 10-12** **Form: $\boxed{\text{T}}$**

Bubble in the single best choice for each question you choose to answer.

1. Using a scale, you find that four tweezers and a bar of soap weigh the same as three combs; a bar of soap weighs the same as two toothbrushes and a comb; and six tweezers weigh the same as one toothbrush and one comb. How many combs are needed to equal the weight of one bar of soap?

   (A) 2
   (B) 4
   (C) 6
   (D) 8
   (E) 10

2. On one moving sidewalk in an airport it takes 2 min to stand the entire ride. Jamie can walk the same distance in 90 s. How long would it take Jamie to walk from start to end on the moving sidewalk?

   (A) 51 s
   (B) 47 s
   (C) 65 s
   (D) 58 s
   (E) 55 s

3. A fountain spouts water from two eyes, a mouth, and a foot. The right eye would fill a specific jar in two days, the left eye in three days, the mouth in six hours, and the foot in four days. To the nearest hour, how long will it take all four of these spouts together to fill the jar?

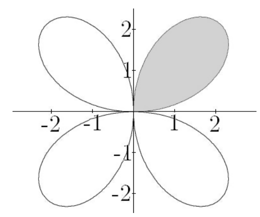
   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

4. What is the slope of the line tangent to $y = x^3$ at $(0, 0)$?

   (A) $\frac{\pi}{4}$
   (B) $\frac{7}{4}$
   (C) $-\frac{7}{4}$
   (D) 0
   (E) -1

5. Find the area inside one leaf of the curve $r = 3\sin(2\theta)$.

   (A) $\frac{9\pi}{8}$
   (B) $\frac{3\pi}{2}$
   (C) $\frac{3\pi}{4}$
   (D) $\frac{9\pi}{4}$
   (E) π



6. Consider the equation $x^2 + bx + 2 = 0$. A single, fair, 6-sided die is rolled to determine the value of the middle coefficient b which becomes the number of pips on the upper face of the die. What is the probability that the equation will have real unequal roots?

   (A) $\frac{1}{6}$
   (B) $\frac{1}{3}$
   (C) $\frac{1}{2}$
   (D) $\frac{2}{3}$
   (E) $\frac{5}{6}$

7. The base three representation of x is 12112211122211112222. Find the left digit of the base nine representation of x.
   - (A) 1
   - (B) 2
   - (C) 3
   - (D) 4
   - (E) 5

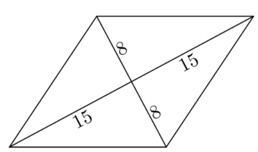
8. What is the value of the expression?

$$\sqrt{16 + \sqrt{16 + \sqrt{16 + \dots}}}$$

   - (A) $2\sqrt{2}$
   - (B) 4
   - (C) 4.52
   - (D) $-\frac{1}{2} + \frac{\sqrt{65}}{2}$
   - (E) $\frac{1}{2} + \frac{\sqrt{65}}{2}$

9. If $\log_4(\log_4(\log_4(\log_4(x)))) = 0$, what is x?
   - (A) 1
   - (B) 256^3
   - (C) 4^{16}
   - (D) 2^{512}
   - (E) 256^4

10. Find the area and perimeter of the rhombus.
   - (A) $A = 480, P = 88$
   - (B) $A = 289, P = 68$
   - (C) $A = 240, P = 68$
   - (D) $A = 225, P = 60$
   - (E) $A = 120, P = 40$



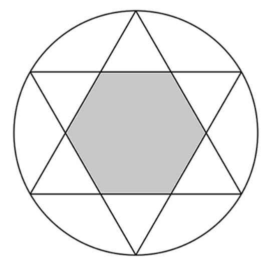
11. If $a + b = 3$ and $a^2 + b^2 = 89$, then what is $a^3 + b^3$?
   - (A) 307
   - (B) 347
   - (C) 387
   - (D) 507
   - (E) Not possible to determine

12. How many ordered triples (x, y, z) of real numbers satisfy the conditions?

$$xy = z, \;\; xz = y, \;\; yz = x$$

   - (A) 1
   - (B) 2
   - (C) 3
   - (D) 4
   - (E) 5

13. Two equilateral triangles are arranged into a perfectly symmetrical "Star of David" and circumscribed by a circle of radius 1. What is the area of the regular hexagon formed by the intersection of the two triangles?
   - (A) 1
   - (B) $\frac{\sqrt{3}}{2}$
   - (C) $\frac{\pi}{2}$
   - (D) $\sqrt{3}$
   - (E) π



14. $\frac{68}{77}$ can be expressed as the sum of two positive proper fractions whose denominators are 7 and 11: $\frac{a}{7} + \frac{b}{11}$. What is $a + b$?
   - (A) 6
   - (B) 8
   - (C) 11
   - (D) 17
   - (E) $8\sqrt{5}$

15. If $\sin x = 3 \cos x$, what is $\sin x \cos x$?
   - (A) $\frac{1}{3}$
   - (B) $\frac{3}{10}$
   - (C) $\frac{2}{3}$
   - (D) $\frac{5}{6}$
   - (E) $\frac{3}{2}$

16. One hundred tickets numbered 1 through 100 are in a bowl. The tickets are drawn one at a time without replacement. How many tickets must be drawn to be sure that one of the selected numbers is exactly twice as large as another selected number?

    (A) 50
    (B) 63
    (C) 68
    (D) 75
    (E) 84

17. In 6 years Bill's age will be a perfect square. Six years ago Bill's age was the square root of that perfect square. How old is Bill?

    (A) 6
    (B) 8
    (C) 10
    (D) 12
    (E) 14

18. Antoine, Benoît, Claude, Didier, Étienne, and François go to the cinema. They want to sit in a single row of six seats, but A, B, and C are mutual enemies and refuse to sit next to either of the other two. How many different arrangements are possible?

    (A) 169
    (B) 121
    (C) 136
    (D) 144
    (E) 125

19. A and B want to know when C's birthday is. C gives them a list of 10 possible dates.

May 15	May 16	May 19
June 17	June 18	
July 14	July 16	
August 14	August 15	August 17

    C then tells A and B separately the month and the day of her birthday respectively.

    A: I don't know when C's birthday is, but I know that B does not know either.
    B: At first I didn't know when C's birthday is, but I know now.
    A: Then I also know when C's birthday is.

    So when is C's birthday?

    (A) May 16
    (B) May 19
    (C) June 17
    (D) July 14
    (E) July 16

20. A merchant buys some kiwis at 3 for 10¢ and an equal number at 5 for 20¢. What should the selling price be to "break even"?

    (A) 8 for 30¢
    (B) 3 for 11¢
    (C) 5 for 18¢
    (D) 11 for 40¢
    (E) 13 for 50¢

21. A golden triangle is an isosceles triangle in which the ratio of the duplicated side to the distinct side is $\phi = (1 + \sqrt{5})/2$. Unique to golden triangles is that the three angles have a 2:2:1 proportion. How many non-overlapping golden triangles can be arranged around a common vertex?

    (A) 10
    (B) 12
    (C) 15
    (D) 18
    (E) 20

22. The strength of the electric force between two particles with charge q_1 and q_2 a distance r apart is

$$F = k\frac{|q_1|\,|q_2|}{r^2}$$

If the distance between two charged particles is trebled (tripled), what is the ratio of the new force to the original force?

(A) 9:1

(B) 3:1

(C) 1:1

(D) 1:3

(E) 1:9

23. The simplest polygon in a plane is a triangle (whose angles sum to 180°). On a sphere the simplest polygon is a *biangle* (or *lune*). What is the range for the sum S of the angles in a biangle confined to one hemisphere?

(A) $0° < S < 360°$

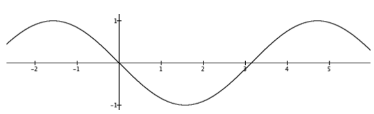
(B) $45° < S < 360°$

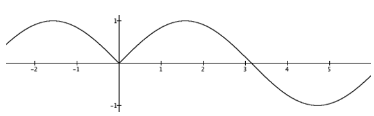
(C) $45° < S < 180°$

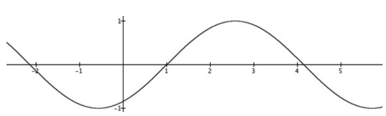
(D) $0° < S < 180°$

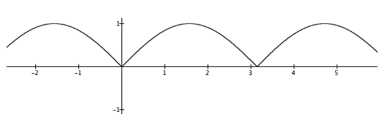
(E) $90° < S < 180°$

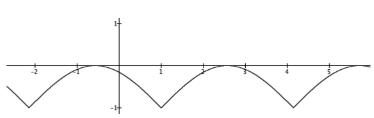
24. Given $f(x) = |x-1| - 1$ and $g(x) = \sin x$, which is a graph of the composition of the two functions, $(f \circ g)(x) = f(g(x))$?

(A) 

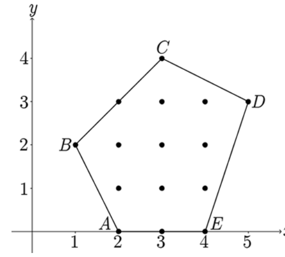
(B) 

(C) 

(D) 

(E) 

25. Let $A = (2,0)$, $B = (1,2)$, $C = (3,4)$, $D = (5,3)$, and $E = (4,0)$. Find the area, in square units, of pentagon $ABCDE$.



(A) 10.5

(B) 12

(C) 9

(D) 10

(E) 11.5

26. Flip a fair coin 10 times. What is the probability that at least 3 heads appear?

(A) $\frac{968}{1024}$

(B) $\frac{992}{1024}$

(C) $\frac{946}{1024}$

(D) $\frac{1013}{1024}$

(E) $\frac{923}{1024}$

27. On what interval(s) is the graph of $h(x) = x^3 - x^2 + 5$ concave up?

(A) $(-\infty, 0)$ and $(\frac{2}{3}, \infty)$

(B) $(-\infty, \frac{1}{3})$

(C) $(3, \infty)$

(D) $(\frac{1}{3}, \infty)$

(E) $(\frac{1}{3}, \frac{2}{3})$

28. Given: a, b, c, d are non-zero real numbers and $a + b\mathrm{i}$ and c are roots of the polynomial $x^4 + dx^3 + dx + 1$ (a palindromic polynomial because of the symmetry of the coefficients). What is the sum of all real roots?

(A) $2a$

(B) $\frac{c^2+1}{c}$

(C) $a + c$

(D) $2a + 2c$

(E) $2c$

29. A *fixed point* of a set S_n of permutations (or reorderings) of n objects is a number in the list $\{1, 2, \ldots, n\}$ that the permutation leaves alone. For example, $1 \mapsto 2$, $2 \mapsto 1$, $3 \mapsto 3$ is a reordering of $\{1, 2, 3\}$ to the permutation $\{2, 1, 3\}$ which leaves 3 alone. For $n = 4$ how large is the largest subset of permutations in S_4 such that every permutation in that subset shares the same fixed point?

(A) 1
(B) 3
(C) 4
(D) 6
(E) 12

30. What is the smallest integer that can be written in two different ways (different numbers) as the sum of two positive cubes?

(A) 341
(B) 728
(C) 1339
(D) 1729
(E) 4104

31. What x-values satisfy $\sqrt{x^2 + y^2} = 5 - x$?

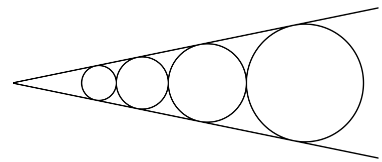
(A) $x \in (-\infty, \infty)$
(B) $x \in \left(-\infty, \frac{5}{2}\right]$
(C) $x \in [-5, 5]$
(D) $x = 0$
(E) $x \in \left[\frac{5}{2}, 5\right]$

32. What is the smallest positive integer n such that $1 + 2 + 3 + \ldots + n > 5000$?

(A) 90
(B) 99
(C) 100
(D) 101
(E) 110

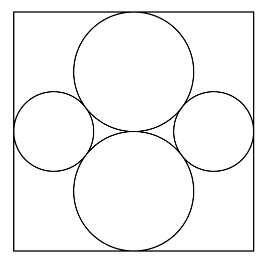
33. Four circles are placed in an angle so that they all just fit as shown. If the radius of the smallest is 16 and the radius of the largest is 54, what are the radii of the two circles in between?

(A) $22\frac{2}{3}, 35\frac{1}{3}$
(B) $24, 36$
(C) $27, 42$
(D) $28\frac{2}{3}, 39\frac{1}{3}$
(E) $30, 45$



34. In the figure all four circles are tangent to the square, the two large circles have the same radius and are tangent to each other, and the two small circles have the same radius and are tangent to each of the large circles. If the radius of the large circles is 1, what is the radius r of the small circles?

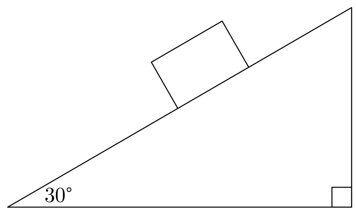
(A) $\frac{1}{2}$
(B) $\frac{2}{3}$
(C) $\frac{\sqrt{2}}{2}$
(D) $\frac{3}{4}$
(E) $\frac{\sqrt{3}}{2}$



35. Compute the value of the limit. $\lim_{x \to 0} \dfrac{\tan x}{2x}$

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) The limit does not exist

36. Newton's laws tell us that for every force exerted by an object, there is an equal and opposite force exerted back on that object. If a box is held in place on a 30° inclined plane by a static friction force of 125 newtons, what is the magnitude of the force exerted by the plane on the box?



(A) $0\,\mathrm{N}$

(B) $62.5\,\mathrm{N}$

(C) $\frac{125\sqrt{3}}{3}\,\mathrm{N}$

(D) $125\,\mathrm{N}$

(E) $125\sqrt{3}\,\mathrm{N}$

37. Two objects are topologically equivalent if we can stretch, shrink, bend, or twist one, without cutting or gluing, and deform it into the other. Which is equivalent to **F**?

(A) **G**

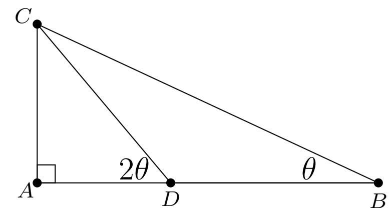
(B) **L**

(C) **W**

(D) **X**

(E) **Y**

38. In triangle ABC, given $m\angle ABC = \theta$ and $m\angle ADC = 2\theta$, which formula correctly gives the length of side AC in terms of the length of side BD and the angle θ?



(A) $AC = BD \tan 3\theta$

(B) $AC = BD \cos \theta$

(C) $AC = BD \cos 2\theta$

(D) $AC = BD \sin \theta$

(E) $AC = BD \sin 2\theta$

39. Evaluate. $(1+\mathrm{i})^2$

(A) $-2-\mathrm{i}$

(B) $-1+\mathrm{i}$

(C) $2\mathrm{i}$

(D) $1-2\mathrm{i}$

(E) $2+2\mathrm{i}$

40. What is the output of the following BASIC computer program?

```
10 for i = 1 to 5
20 print i^i/i
30 next i
```

(A) `1  2  9  64  625`

(B) `1  2  3  4  5`

(C) `1  4  27  256  3125`

(D) `1  4  9  16  25`

(E) `1  32  243  1024  3125`