Snow College Mathematics Contest

key

April 7, 2015 **Senior Division: Grades 10-12** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Chuck, Jackie, and Bruce are all celebrating their birthdays on the same day. Bruce's present age is seven years less than the sum of Chuck's and Jackie's present ages. In five years, Bruce will be twice as old as Jackie will be then. Three years ago, Jackie was one-third as old as Chuck was. What is the sum of all of their current ages?

   (A) 25

   (B) 31

   (C) 39

   (D) 48

   (E) 53

   SOLN Create the following system of equations: (1) $b = (c + j) - 7$, (2) $b+5 = 2(j+5)$, and (3) $j-3 = \frac{1}{3}(c-3)$. From (2) we get $b = 2j + 5$, and from (3) we get $c = 3j - 6$. By substitution (1) becomes $(2j+5) = (3j-6)+j-7$. Solving at this point yields $j = 9$, $c = 21$, and $b = 23$. Sum = 53. □

2. If $x^2 + 1$ is a factor of $6x^3 + 5x^2 + Px + Q$, then what is $P + Q$?

   (A) 10

   (B) 11

   (C) 12

   (D) 13

   (E) 14

   SOLN Factor by grouping:
   $x^2(6x + 5) + (Px + Q)$
   This is $(x^2 + 1)(6x + 5)$
   if $Px + Q = 6x + 5$. □

3. There are 3 cards in a bag; one has X on both sides, one has O on both sides, and the third has X on one side and O on the other. If you draw a card at random and look at one side and see an X, what is the probability that the other side is also X?

   (A) $\frac{1}{4}$

   (B) $\frac{1}{3}$

   (C) $\frac{1}{2}$

   (D) $\frac{2}{5}$

   (E) $\frac{2}{3}$

   SOLN There are three possible Xs you could see on the first side; two of them have X on the other side too. □

4. A rotation in the xy-plane through an angle θ about the origin is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

   What are the coordinates (x', y') of the image of the point $(3, 2)$ after a rotation through $-90°$?

   (A) $(3, 2)$

   (B) $(-3, 2)$

   (C) $(3, -2)$

   (D) $(2, 3)$

   (E) $(2, -3)$

   SOLN If one knows matrix multiplication and a little trig:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

   This can also be found by graphing. Negative angles go clockwise. □

5. Evaluate the product: $\log_2(125) \cdot \log_5(16)$

(A) 200

(B) 20

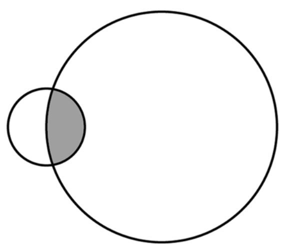
(C) 12

(D) 10

(E) 10.5

SOLN $\log_2(125) = \log_2(5^3) = 3\log_2(5)$
Similarly, $\log_5(16) = 4\log_5(2)$.
Recall the change of base formula: $\log_b(a) \cdot \log_a(b) = 1$
$3\log_2(5) \cdot 4\log_5(2) = 12$ □

6. Two overlapping circles have radii 1 and 3. If the shaded area is $\frac{\pi}{2}$, then what is the total area of the figure?



(A) 10π

(B) $\frac{19\pi}{2}$

(C) 8π

(D) $\frac{7\pi}{2}$

(E) 9π

SOLN $A = \pi(1)^2 + \pi(3)^2 - \frac{\pi}{2}$ □

7. If you place one penny on the first square of an 8×8 checkerboard, two pennies on the second square, four pennies on the third square, and keep doubling the number of pennies for each square, how many total pennies will be on the entire board?

(A) 2^{64}

(B) 2^{63}

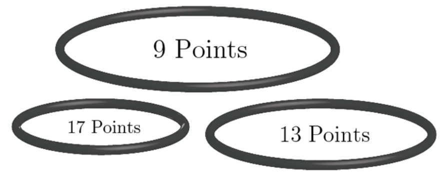
(C) $2^{63} + 2^{62}$

(D) 2^{65}

(E) $2^{64} - 1$

SOLN The number of pennies forms a geometric series $1 + 2 + 4 + \ldots + 2^{63}$. Summing the series yields $2^{64} - 1$. □

8. George and Lucas are playing a game where they toss balls into three hoops. The smallest hoop is worth 17 points, the middle-sized hoop is worth 13 points, and the largest hoop is worth 9 points. George lost with only 84 points. What is the minimum number of balls he must have thrown?



(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

SOLN Case 1: 5 balls in the 9 point hoop and 3 balls in the 13 point hoop.
Case 2: 6 balls in the 9 point hoop, 1 ball in the 17 point hoop, and 1 ball in the 13 point hoop.
For both of these cases, 8 balls were necessary to achieve the score of 84. □

9. Say we call a date mm/dd/yy *magical* if mm $\times$ dd = yy. How many of the following dates can **never** be magical?

- January 31
- February 29
- March 31
- April 30

(A) 0

(B) 1

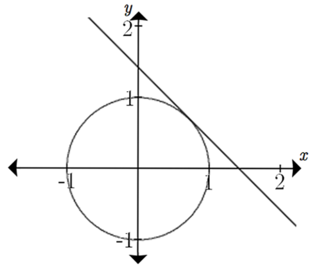
(C) 2

(D) 3

(E) 4

SOLN April 30: 04×30 is not a two-digit number. February 29: $02 \times 29 = 58$ but 58 is not a leap year. □

10. Consider the point on the unit circle with an angle $\theta = \pi/4\,\text{rad}$ (ccw from the $+x$-axis). Where does the line through that point and tangent to the circle intersect the y-axis?



(A) $(0, 0)$

[(B)] $(0, \sqrt{2})$

(C) $(0, 1)$

(D) $(0, 2)$

(E) $(\sqrt{2}, 0)$

SOLN The point is $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. Symmetry says this line has a slope of -1. Now we have the slope of the tangent line and one point it goes through. $y - \frac{\sqrt{2}}{2} = (-1)(x - \frac{\sqrt{2}}{2})$. In slope-intercept form: $y = -x + \sqrt{2}$. □

11. Express the continued fraction expression for x as a simple closed-form number.

$$x = 1 + \cfrac{6}{2 + \cfrac{6}{2 + \cfrac{6}{2 + \cfrac{6}{\ddots}}}}$$

[(A)] $\sqrt{7}$

(B) $\frac{1-\sqrt{2}}{2}$

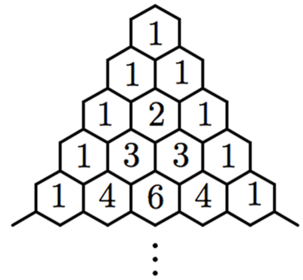
(C) $\frac{1+\sqrt{2}}{2}$

(D) $2 + \frac{\sqrt{2}}{2}$

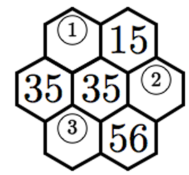
(E) $\frac{4}{\sqrt{2}}$

SOLN Call the fraction part y. See that the whole equation is equivalent to $x = 1 + y = 1 + \frac{6}{2+y}$. So $y = \frac{6}{2+y}$. Rearrange to get $y^2 + 2y - 6 = 0$ and use the quadratic formula. The two solutions are $y = -1 \pm \sqrt{5}$. $x = 1 + y = \pm\sqrt{7}$ but only the positive one equals the original continued fraction. □

12. Here is the start of Pascal's triangle.



Here is another part that would show up further down. Fill in the numbered spots.



(A) 1) 20 2) 1 3) 56

(B) 1) 15 2) 21 3) 70

(C) 1) 15 2) 35 3) 80

[(D)] 1) 20 2) 21 3) 70

(E) 1) 15 2) 35 3) 56

SOLN Each number is the sum of the two above it. □

13. Assume the following statements are true.

- If it rains, we will bake a cake.
- If it doesn't rain, we will play football.
- If we play football, I will get muddy.
- Mother will get angry if I get muddy.
- I did not get muddy.

Which is a valid conclusion?

(A) Mother is not angry.

[(B)] We baked a cake.

(C) We did not bake a cake.

(D) It did not rain.

(E) We played football.

SOLN If I didn't get muddy then we didn't play football, and it rained. □

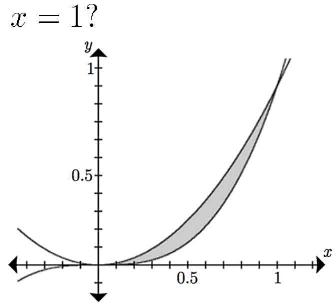
14. When a flock of sheep are driven through Sanpete Sally's land near Ephraim, she charges a toll of 10¢ per riderless animal (i.e., the sheep and dogs) and 50¢ for each rider and horse pair. One day Sally counted a total of 4168 legs (including riders, horses, dogs, and sheep) and 1044 heads. How much money did Sally collect?

- **(A)** \$105.60
- (B) \$123.80
- (C) \$305.20
- (D) \$518.40
- (E) \$961.00

SOLN legs: $2r + 4(h + s + d) = 4168$
heads: $r + (h + s + d) = 1044$
Solve head equation for $(h+s+d)$ and insert into the leg equation.
$2r + 4(1044 - r) = 4168 \implies r = 4$
4−legged animals $= 1044 - 4 = 1040$
$s + d = 1040 - h = 1040 - 4 = 1036$
Total toll $= 1036(\$0.10) + 4(\$0.50) =$ \$105.60 □

15. What is the area between $y = x^2$ and $y = x^3$ between $x = 0$ and $x = 1$?

- **(A)** $\frac{1}{12}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{1}{8}$
- (D) $\frac{2}{5}$
- (E) $\frac{4}{9}$



SOLN $\int_0^1 (x^2 - x^3)\mathrm{d}x = \left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ □

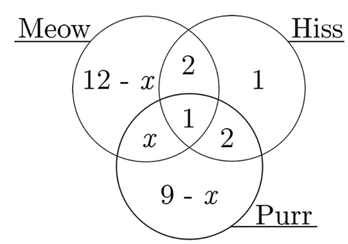
16. If $2^4 + m^n = 2^5$, then what is $m + n$ if m, n are positive integers?

- **(A)** 6 or 17
- (B) 4 or 6
- (C) 5 or 8
- (D) 16 or 17
- (E) 4 or 8

SOLN $2^5 - 2^4 = 16$, so $m^n = 16$.
The positive integer solutions are $\{(m, n)\} = \{(4, 2), (2, 4), (16, 1)\}$. □

17. I only heard my cat meow, hiss, and purr on one day out of the last 23 days. But I did hear him make at least one of these sounds each day. I heard him hiss on 6 days, purr on 12 days, and meow on 15 days. On 2 days, I heard him meow and hiss but not purr, and on 2 days I heard him purr and hiss but not meow. On how many days did I hear him meow and purr but not hiss?

- (A) 3
- **(B)** 4
- (C) 5
- (D) 6
- (E) 7



SOLN The sections of the Venn diagram add to 23. $23 = 1 + 1 + 2 + 2 + x + (9 - x) + (12 - x) = 27 - x \implies x = 4$
Soln 2: Inclusion-exclusion principle. M = days meowed, H = days hissed, and P = days purred. $23 = |M \cup H \cup P| = |M| + |H| + |P| - |M \cap H| - |H \cap P| - |M \cap P| + |M \cap H \cap P|$
$= 15 + 6 + 12 - (2 + 1) - (2 + 1) - (x + 1) + 1 \implies x = 4$ □

18. What is the range of the function?
$f(x) = \sqrt{8\sin^3 x + 17}$

(A) $[0, \infty)$

(B) $[0, 5]$

(C) $[0, 23]$

(D) $[3, 5]$

(E) $[3, 23]$

SOLN The range of $\sin x$ is $[-1, 1]$; the range of that cubed is also $[-1, 1]$. Check left end: $\sqrt{-8+17} = 3$; right end: $\sqrt{8+17} = 5$. □

19. What is the output of the following BASIC computer program?

```
10 s = 0 : rem initialize sum to 0
20 for i = 1 to 3 : rem outer loop
30 for j = 1 to i : rem inner loop
40 s = s+j : rem add j to old s
50 next j : rem inner loop
60 next i : rem outer loop
70 print s : rem print the sum
```

(A) 0

(B) 4

(C) 10

(D) 15

(E) 20

SOLN Nested loops. When `i` $= 1$, `j` will only be 1 and `s` is 1. When `i` $= 2$, `j` adds a 1 and a 2 and `s` is 4. When `i` $= 3$, `j` adds a 1, a 2, and a 3. □

20. Given same-size square matrices A and B, when is $(A+B)^2 = A^2 + 2AB + B^2$?

(A) always

(B) never

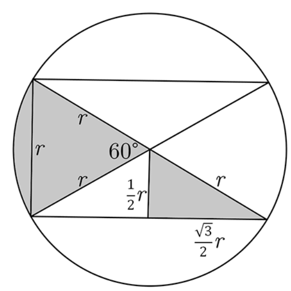
(C) only if A or B is the identity

(D) only if A and B commute

(E) only if A and B are upper triangular

SOLN $(A+B)(A+B) = A^2 + AB + BA + B^2$. The middle terms are the same iff the matrices commute. □

21. Two congruent parallel chords are drawn a distance r apart in a circle of radius r. What is the area of the part of the circle that lies between the chords?



(A) $\left(\frac{\sqrt{3}}{3} + \frac{\pi}{2}\right) r^2$

(B) $\left(\frac{\sqrt{3}}{3} + \frac{\pi}{3}\right) r^2$

(C) $\left(\frac{\sqrt{3}}{2} + \frac{\pi}{2}\right) r^2$

(D) $\left(\frac{\sqrt{2}}{3} + \frac{\pi}{3}\right) r^2$

(E) $\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) r^2$

SOLN $A_{\text{wedge}} = \frac{1}{6} A_{\bigcirc} = \frac{1}{6}\pi r^2$

$A_{\triangle} = \frac{1}{2}bh = \frac{1}{2}\left(\frac{\sqrt{3}}{2}r\right)\left(\frac{1}{2}r\right) = \frac{\sqrt{3}}{8}r^2$

$A_{\text{tot}} = 4A_{\triangle} + 2A_{\text{wedge}}$ □

22. If $f(x) = 2x+3$ and $g(x) = x^2 - 5$, what is $(f \circ g)(3)$?

(A) 11

(B) 15

(C) 19

(D) 23

(E) 76

SOLN $(f \circ g)(3) = f(g(3)) = f(4)$ □

23. Which of these is a perfect square in every possible base b for that number?
(e.g., 321_b is only a number when $b > 3$).

(A) 36_b

(B) 81_b

(C) 144_b

(D) 225_b

(E) 3993_b

SOLN $1(b^2) + 4(b) + 4 = b^2 + 4b + 4 = (b+2)^2$ which is a perfect square □

24. If Bert had three more quarters, he would have twice as many quarters as Ernie. If Ernie had six more quarters, he would have three times as many quarters as Bert. If Bert and Ernie put their quarters together, how many quarters would they have?

(A) 6

(B) 8

(C) 10

(D) 12

(E) 16

SOLN $B + 3 = 2E \quad E + 6 = 3B$
$E+6 = 3(2E-3) \implies E+6 = 6E-9$
$-5E = -15 \implies E = 3, \quad B = 3$ □

25. If $\frac{a}{b+c+d} = \frac{4}{3}$ and $\frac{a}{b+c} = \frac{3}{5}$, then what is the value of $\frac{d}{a}$?

(A) $\frac{7}{6}$

(B) $\frac{6}{7}$

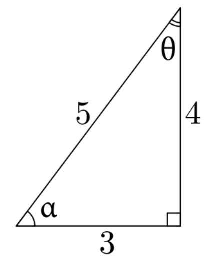
(C) $-\frac{12}{11}$

(D) $-\frac{11}{12}$

(E) $\frac{15}{11}$

SOLN Flip first: $\frac{3}{4} = \frac{b+c+d}{a} = \frac{b+c}{a} + \frac{d}{a}$
$\frac{d}{a} = \frac{3}{4} - \frac{b+c}{a} = \frac{3}{4} - \frac{5}{3} = -\frac{11}{12}$ □

26. If $3\sin\theta + 4\cos\theta = 5$, then what is $\tan\theta$?



(A) 1

(B) -1

(C) $\frac{4}{3}$

(D) $\frac{3}{4}$

(E) 0

SOLN $\frac{3}{5}\sin\theta + \frac{4}{5}\cos\theta = \frac{5}{5} = 1$. See figure: $\cos\alpha = \frac{3}{5}$ and $\sin\alpha = \frac{4}{5}$ and we have $\cos\alpha\sin\theta + \sin\alpha\cos\theta = 1$. Use addition identity: $\sin(\alpha+\theta) = 1$ so α and θ are complements and the upper angle in the figure is θ. □

27. If the line $y = mx+k$ intersects the parabola $y = ax^2 + bx + c$ at two points, what is the product of the two x-coordinates of these points in terms of a, b, c, m, and k?

(A) $\frac{c+k}{a}$

(B) $\frac{ck}{a}$

(C) $\frac{c+k}{b}$

(D) $\frac{c-k}{b}$

(E) $\frac{c-k}{a}$

SOLN $mx + k = ax^2 + bx + c \implies ax^2 + (b-m)x + (c-k) = 0$

$$x = \frac{-(b-m) \pm \sqrt{(b-m)^2 - 4a(c-k)}}{2a}$$

Since solutions are conjugates $x_1 \cdot x_2 =$

$$\frac{(b-m)^2 - [(b-m)^2 - 4a(c-k)]}{4a^2}$$

$$= (c-k)/a$$ □

28. If $x < 0$, then what is $\left|x - \sqrt{(x-1)^2}\right|$?

(A) 1

(B) $-2x - 1$

(C) $1 + 2x$

[(D)] $1 - 2x$

(E) $2x - 1$

SOLN $\left|x - \sqrt{(x-1)^2}\right| = |x - |x-1||$. $x < 0 \Rightarrow |x-1| = -(x-1) = -x+1$. $|x - |x-1|| = |x - (-x+1)| = |2x-1|$ $= -(2x-1) = 1 - 2x$ □

29. Solve for y in the figure.

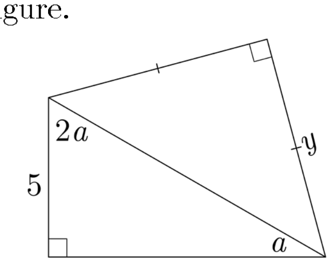
(A) 5

[(B)] $5\sqrt{2}$

(C) $5\sqrt{3}$

(D) 10

(E) $2\sqrt{5}$



SOLN In a 30-60-90 triangle the long leg is $\sqrt{3}$ times the short leg, so $x = 5\sqrt{3}$. The hypotenuse is twice the short leg, so it is 10. In a 45-45-90 triangle the hypotenuse is $\sqrt{2}$ times a leg, so $y = 5\sqrt{2}$ since $5\sqrt{2}(\sqrt{2}) = 10$. □

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x+y) = f(xy)$ for all real x, y, and $f(7) = 7$. What is $f(49)$?

(A) 1

[(B)] 7

(C) 14

(D) 21

(E) 49

SOLN $x = 0 \implies f(y) = f(0) \implies f(y) = \text{constant}$.
$y = 1 \implies f(x+1) = f(x) \implies f(8) = f(7) \implies f(x) = 7$ □

31. The base three representation of x is 121122111222. Find the leftmost digit of the base nine representation of x.

(A) 1

(B) 2

(C) 3

(D) 4

[(E)] 5

SOLN $1 \cdot 3^{11} + 2 \cdot 3^{10} + 1 \cdot 3^9 + 1 \cdot 3^8 + \ldots + 1 \cdot 3^3 + 2 \cdot 3^2 + 2 \cdot 3^1 + 2 \cdot 3^0$
$= (1 \cdot 3 + 2)3^{10} + (1 \cdot 3 + 1)3^8 + \ldots + (1 \cdot 3 + 2)3^2 + (2 \cdot 3 + 2)3^0$
$= \mathbf{5} \cdot 9^5 + 4 \cdot 9^4 + \ldots + 5 \cdot 9^1 + 8 \cdot 9^0$
Soln 2: x has even # digits and $3^2 = 9$, so each pair of digits base three is one digit in base nine. $12_{\text{three}} = 5_{\text{nine}}$ □

32. A box contains 2 pennies, 4 nickels, and 6 dimes. Six coins are drawn at random without replacement, with each coin having equal probability of being chosen. What is the probability the value of the coins drawn is at least 50¢?

(A) $\frac{37}{924}$

(B) $\frac{91}{924}$

[(C)] $\frac{127}{924}$

(D) $\frac{132}{924}$

(E) $\frac{194}{924}$

SOLN Total number of combinations is $\binom{12}{6} = \frac{12!}{6!6!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6!} = 924$
Ways to get at least 50¢:
- 6 dimes: $\binom{6}{6} = 1$
- 5 dimes + one other: $\binom{6}{5}\binom{6}{1} = 36$
- 4 dimes, 2 nickels: $\binom{6}{4}\binom{4}{2} = 90$
- 3 dimes + three others isn't enough

$1 + 36 + 90 = 127$ □

33. A triangle has side $a = \sqrt{7}$, the opposite angle $\alpha = 60°$, and the sum of the other two sides $b + c = 5$. Find the ratio of the longest to the shortest side of the triangle.

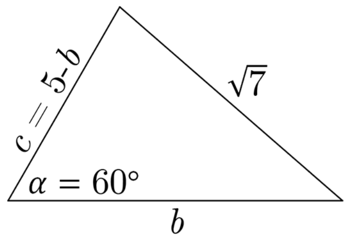
(A) 1

(B) $\sqrt{2}$

[(C)] $\frac{3}{2}$

(D) $\frac{\sqrt{7}}{2}$

(E) 2



SOLN Use law of cosines. $(\sqrt{7})^2 = b^2 + (5-b)^2 - 2(b)(5-b)\cos 60°$
$7 = b^2 + 25 - 10b + b^2 - 2b(5-b)(\frac{1}{2})$
$0 = 3b^2 - 15b + 18 = 3(b-2)(b-3)$
$b = 2, 3 \implies c = 3, 2$
so ratio of long to short is 3/2. □

34. A six-sided die has faces labeled 1 though 6. It is weighted so that a three is three times as likely to be rolled as a one, a three and a six are equally likely, and a one, a two, a four, and a five are equally likely. What is the probability of rolling a three?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

[(D)] $\frac{3}{10}$

(E) $\frac{2}{5}$

SOLN $P(3) = 3P(1)$, $P(3) = P(6)$,
$P(1) = P(2) = P(4) = P(5)$
$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$ $\frac{1}{3}P(3) + \frac{1}{3}P(3) + P(3) + \frac{1}{3}P(3) + \frac{1}{3}P(3) + P(6) = 1$
$\frac{10}{3}P(3) = 1 \implies P(3) = \frac{3}{10}$ □

35. Find the area of a regular hexagon inscribed in a circle with radius 2.

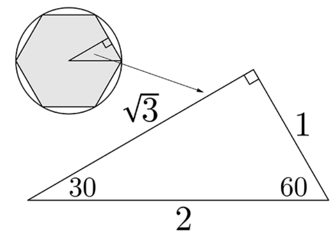
[(A)] $6\sqrt{3}$

(B) 12

(C) $3\sqrt{3}$

(D) $12\sqrt{3}$

(E) 6



SOLN A perpendicular line segment from the circle center to a side (apothem) of the hexagon forms a 30-60-90 triangle. Since the hypotenuse is 2 the short leg is 1 and the long leg (apothem) is $\sqrt{3}$. Since the short leg is 1 the length of a side is 2 (the same length as the radius). The area of a regular polygon is half the apothem times the perimeter: $\frac{12\sqrt{3}}{2}$. □

36. For a closed rectangular box $L + W + H = 25\,\text{cm}$. The surface area of the box is $225\,\text{cm}^2$. Find the distance between opposite corners of the box; i.e., the largest distance between points on the box.

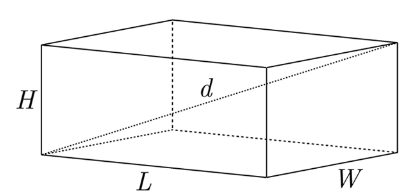
(A) 12.5 cm

(B) 15 cm

[(C)] 20 cm

(D) 25 cm

(E) 27.5 cm



SOLN $d = \sqrt{L^2 + W^2 + H^2}$. Since $(L + W + H)^2 = L^2 + W^2 + H^2 + 2LW + 2LH + 2WH = d^2 + 625$, we have $625 = d^2 + 225 \implies d = 20$. □

37. Consider three circles of radius 1, each tangent to the others. What is the radius of a fourth smaller circle in the middle which is tangent to each of them?

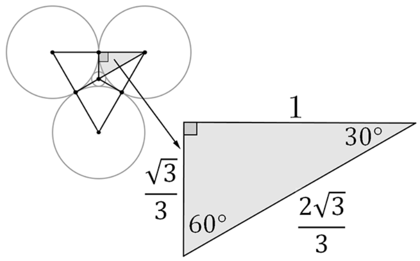
(A) $\frac{1}{4}$

[(B)] $\frac{2\sqrt{3}-3}{3}$

(C) $\frac{\sqrt{3}-1}{2}$

(D) $\frac{\sqrt{3}-1}{4}$

(E) $\frac{\sqrt{3}}{3}$



[SOLN] Drawing line segments as shown, we can create at 30-60-90 triangle with hypotenuse of length $2\sqrt{3}/3$. Since this hypotenuse is the distance between the center of the small circle and the center of a larger circle, the deisred result will be $2\sqrt{3}/3 - 1$. □

38. Suppose f is a linear function such that $(f \circ f \circ f)(x) = 27x + 26$. Find the y-intercept for the graph of f.

[(A)] $(0, 2)$

(B) $(26, 0)$

(C) $(0, 26)$

(D) $(0, 13)$

(E) $(0, 3)$

[SOLN] $f(x) = mx + b \implies (f \circ f)(x) = m(mx + b) + b = m^2x + bm + b \implies (f \circ f \circ f)(x) = m(m^2x + bm + b) + b = m^3x + bm^2 + bm + b$. Equating this to $27x + 26$ we see $m = 3$ so that $27x + 26 = 27x + 13b \Rightarrow b = 2$ □

39. A parallelogram with an area of 580 has vertices $(0, 0)$, $(5, 30)$, and $(20, a)$. If $0 < a < 100$, then what is the value of a?

(A) 3

[(B)] 4

(C) 6

(D) 7

(E) 35

[SOLN] Let $\vec{u} = \langle 20, a \rangle$, $\vec{v} = \langle 5, 30 \rangle$. Then regardless of whether $\vec{u}$ and $\vec{v}$ are diagonals or sides of the parallelogram, $|\vec{u} \times \vec{v}| = 580$ (since $|\vec{u} \times \vec{v}| = |(\vec{u} + \vec{v}) \times \vec{v}| = |\vec{u} \times (\vec{u} + \vec{v})|$.)

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 20 & a & 0 \\ 5 & 30 & 0 \end{vmatrix}$$

$|\vec{u} \times \vec{v}| = |600 - 5a| = 580 \implies a = 4$ or 236. □

40. A college math class has N teaching assistants. It takes the assistants 5 hours to grade homework assignments. One day, another teaching assistant joins them in grading, and all the assignments take only 4 hours to grade. Assuming everyone did the same amount of work, compute the number of hours it would take 1 teaching assistant to grade all the homework assignments.

[(A)] 20

(B) 22

(C) 24

(D) 26

(E) 28

[SOLN] Let W be the work it takes to grade all the assignments. We have $\frac{W}{N} = 5$, $\frac{W}{N+1} = 4$. $5N = 4N + 4 \implies N = 4$. If it takes 4 TAs 5 hours, then it takes 1 TA $4 \cdot 5$ hours. □