Snow College Mathematics Contest

key

April 1, 2008 **Senior division: grades 10-12** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. What is the contrapositive of "If 2 is even, then 4 is even." ?

   (A) "If 2 is odd, then 4 is odd."

   (B) "If 4 is even, then 2 is even."

   (C) "If 4 is even, then 2 is odd."

   $\boxed{\text{(D)}}$ "If 4 is odd, then 2 is odd."

   (E) "If 2 is even, then 4 is odd."

   $\boxed{\text{SOLN}}$ The contrapositive of "$p \to q$" is "$(\neg q) \to (\neg p)$". The contrapositive has the same truth value as the original implication. □

2. Given that $a = 1/x$, $b = 9a$, $c = 1/b$, $d = 9c$, $e = 1/d$, and a, b, c, and d are all distinct non-zero numbers, then which must equal x?

   (A) a

   (B) b

   (C) c

   $\boxed{\text{(D)}}$ d

   (E) e

   $\boxed{\text{SOLN}}$ $x = 1/a = 1/(b/9) = 9/b = 9c = 9(d/9) = d$ □

3. What is the minimum value of the function $f(x) = 4x^2 - 8x + 15$?

   (A) 1

   $\boxed{\text{(B)}}$ 11

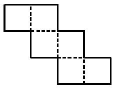
   (C) 15

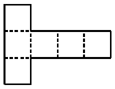
   (D) 27

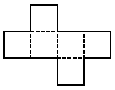
   (E) None of the above

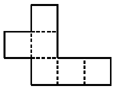
   $\boxed{\text{SOLN}}$ Set the derivative equal to zero and find the functional value at that x. $f'(x) = 8x - 8 = 0 \Rightarrow x = 1$, $f(1) = 11$.
   Or, without calculus, complete the square to put $f(x)$ into vertex form: $f(x) = a(x - h)^2 + k$. Doing so yields $f(x) = 4(x-1)^2+11$, so the minimum is the point $(h, k) = (1, 11)$. □

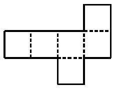
4. Which of the following patterns below CANNOT be folded along the dashed lines to form a cube?

   (A) 

   (B) 

   (C) 

   $\boxed{\text{(D)}}$ 

   (E) 

   $\boxed{\text{SOLN}}$ The topmost and rightmost flaps would be the same side of the cube. □

5. Simplify $(\sqrt{2})^{\log_2 9}$.

$\boxed{\text{(A)}}$ 3

(B) 4.5

(C) 6

(D) 7.5

(E) 9

$\boxed{\mathcal{SOLN}}$ $(\sqrt{2})^{\log_2 9} = (\sqrt{2})^{2\log_2 3} = 2^{\log_2 3} = 3$ □

6. A closed operational system $(G, *)$ is a *group* iff (if and only if)

(i) $(a * b) * c = a * (b * c)$ for every $a, b, c \in G$; [Associativity]

(ii) There exists an element $e \in G$ such that $e * a = a$ and $a * e = a$ for every $a \in G$; [Identity Element]

(iii) For each $a \in G$, there exists an element a^{-1} such that $a * a^{-1} = e$ and $a^{-1} * a = e$. [Inverse Element]

Is the operational system below a group?

$*$	α	β	γ
α	α	β	γ
β	β	α	α
γ	γ	α	β

(A) Yes

$\boxed{\text{(B)}}$ No, because (i) is violated.

(C) No, because (ii) is violated.

(D) No, because (iii) is violated.

(E) No, because two are violated.

$\boxed{\mathcal{SOLN}}$ There is an identity element, α, and each element has at least one inverse; but the operational system is not associative: $(\gamma*\beta)*\beta \neq \gamma*(\beta*\beta)$. Interesting notes: (1) This system is commutative (the table is symmetric across the main diagonal), and (2) In a group the inverse of each element is unique; in this system β is its own inverse, but γ is also an inverse of β. □

7. Ed bought clothes for school. In the first store he spent half his money plus \$10. In the second store he spent half of what was left plus \$5. In the last store he spent three-fourths of what was left and came home with \$5. How much did he start out with?

$\boxed{\text{(A)}}$ \$120

(B) \$80

(C) \$60

(D) \$40

(E) \$30

$\boxed{\mathcal{SOLN}}$ Work backward:
$4 \times \$5 = \$20$
$2 \times (\$20 + \$5) = \$50$
$2 \times (\$50 + \$10) = \$120$ □

8. Seven friends are sitting in a theater watching a show. The row they are all in contains exactly seven seats. After intermission, they return to the same row but choose seats randomly. What is the probability that neither of the people sitting in the two aisle seats was previously sitting in an aisle seat?

(A) 3/7

$\boxed{\text{(B)}}$ 10/21

(C) 11/21

(D) 4/7

(E) 25/49

$\boxed{\mathcal{SOLN}}$ If A and B are the two people who initially sat on the aisle, then after intermission the probability of A sitting in a non-aisle seat is 5/7. If A sits in a non-aisle seat then the probability of B sitting in a non-aisle seat is 4/6. The probability of both events (A and B sitting in non-aisle seats) is $\frac{5}{7} \cdot \frac{4}{6} = \frac{10}{21}$. □

9. A square has four corners (or vertices), four edges, and one face. A cube has eight corners, twelve edges, six faces, and one volume. How many faces would a 4-D hypercube (*a.k.a.*, a tesseract) have?

$\boxed{\text{(A)}}$ 24

(B) 28

(C) 30

(D) 32

(E) 36

$\boxed{\text{SOLN}}$ The number of faces in dimension $n+1$ is $2f+e$ where f and e are the number of faces and edges in dimension n. $\square$

10. How many times does a clock strike in one day if it strikes the time on each hour and once on each half hour? (e.g., 2:00 = two dings, 2:30 = one ding, 3:00 = three dings, 3:30 = one ding.)

(A) 24

(B) 48

(C) 90

$\boxed{\text{(D)}}$ 180

(E) 210

$\boxed{\text{SOLN}}$ From 12:01 to 12:00, the clock will strike once for 1:00, twice for 2:00, etc. That is, for the hours, $1+2+3+4+5+6+7+8+9+10+11+12$. These numbers form an arithmetic sequence whose sum is $(1+12)(12/2) = 13\times 6 = 78$. Adding 12 for the half hours gives 90. Since one day goes through the face of the clock twice we must multiply by 2. $90 \times 2 = 180$. $\square$

11. How many squares (of any size from 1×1 to 8×8) are there on a checker board (an 8×8 grid)?

(A) 40320

(B) 64

$\boxed{\text{(C)}}$ 204

(D) 85

(E) 144

$\boxed{\text{SOLN}}$ There are

$$\begin{array}{l} 64\ (=8^2)\ 1\times 1 \text{ squares,}\\ 49\ (=7^2)\ 2\times 2 \text{ squares,}\\ 36\ (=6^2)\ 3\times 3 \text{ squares,}\\ 25\ (=5^2)\ 4\times 4 \text{ squares,}\\ 16\ (=4^2)\ 5\times 5 \text{ squares,}\\ 9\ (=3^2)\ 6\times 6 \text{ squares,}\\ 4\ (=2^2)\ 7\times 7 \text{ squares,}\\ \underline{1\ (=1^2)\ 8\times 8 \text{ square}}\\ 204 \text{ total} \end{array}$$ $\square$

12. If $\sin(\theta) = 0.5$ then which of the following could be the value of $\cos(\theta)$?

(A) 0.5

(B) 0

(C) -0.5

$\boxed{\text{(D)}}$ $\sqrt{3}/2$

(E) $\sqrt{2}/2$

$\boxed{\text{SOLN}}$ If $\sin(\theta) = 0.5$ then $\theta = \pi/6$ or $\theta = 5\pi/6$. Then $\cos(\theta) = \pm\sqrt{3}/2$. $\square$

13. Simplify $2^{2008} + 2^{2008}$.

(A) 4^{2008}

(B) 4^{4016}

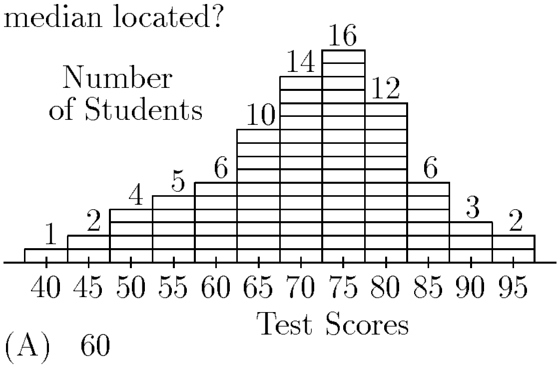
$\boxed{\text{(C)}}$ 2^{2009}

(D) 2^{4016}

(E) None of these

$\boxed{\text{SOLN}}$ $2^{2008} + 2^{2008} = 2(2^{2008}) = 2^{2009}$ $\square$

14. Consider this histogram of scores for 81 students taking a test. In which interval is the median located?



(A) 60

(B) 65

[✓ (C)] 70

(D) 75

(E) 80

SOLN Count cumulatively from either direction. The 41st score is the median score, and it occurs in the interval labeled 70. □

15. The vector cross product is defined as

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

What is $(3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}) \times (1\hat{\imath} + 2\hat{\jmath} - 1\hat{k})$?

(A) $(-6\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$

(B) $(6\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$

(C) $(6\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

(D) $(-6\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

[✓ (E)] $(6\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

SOLN Use the cofactor expansion on the first row to find the determinant.

$$\begin{vmatrix} 4 & -5 \\ 2 & -1 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 3 & -5 \\ 1 & -1 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} \hat{k}$$

The cross product is perpendicular to both $\vec{a}$ and $\vec{b}$. □

16. Consider $f(x) = x^3 - x + c$. When $c = 0$ there are three real roots (you can verify this by setting $c = 0$ and factoring the left side of $x^3 - x = 0$). Characterize the roots when $c = 1$.

(A) two real; one complex

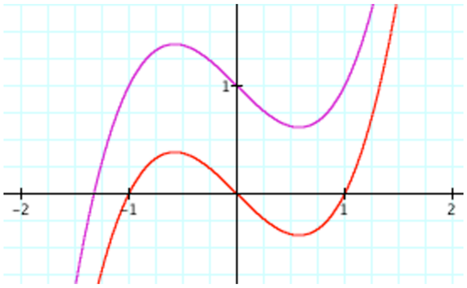
[✓ (B)] one real; two complex

(C) three real

(D) three complex

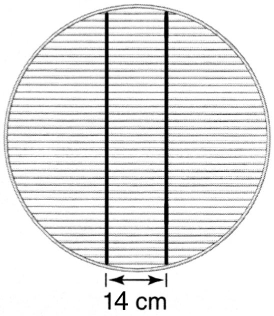
(E) None of the above

SOLN Complex roots of a polynomial, if any, *must* come in pairs. The local minimum of $f(x)$ occurs at $x = \sqrt{3}/3$ but when $c = 1$, $f(\sqrt{3}/3)$ is positive, so $f(x)$ can cross the x-axis only once.



□

17. In the BBQ grill (diameter 50 cm) pictured, the two parallel support rods are equidistant from the center. What is the length of one of them?



(A) 46 cm

(B) 47 cm

[✓ (C)] 48 cm

(D) 49 cm

(E) 50 cm

SOLN Draw a radius from the center of the grill to the end of a support rod. Use the Pythagorean theorem on the right triangle formed by the radius, half the support rod (call that x), and 7 cm. $25^2 - 7^2 = x^2$. Then $L = 2x$. □

18. A community group has 500 people. At the April 1 dance, new members pay only \$14, whereas longtime members pay \$20. As a result, all of the new members attend, but only 70% of the longtime members attend. How much revenue is collected?

$\boxed{\text{(A)}}$ \$7000

(B) \$6000

(C) \$5800

(D) \$5400

(E) There is not enough information.

$\boxed{\text{SOLN}}$ You don't have to know how many of the 500 members are new, but this is only true for the particular numbers that were chosen, not in general. It works because \$20(0.7) = \$14. Let R be the ticket revenue, N be the number of new members, L be the number of longtime members, and T be the total number of members (500 in this case).

$R = \$14(N) + \$20(0.7)(L)$

$R = \$14(N) + \$20(0.7)(T - N)$

$R = \$14(N) + \$14(T - N)$

$R = \$14(N + T - N)$

$R = \$14(T)$

$R = \$14(500) = \$7000$ □

19. Given the scoring formula Score $= 4R - W$, where R and W are the number right and wrong respectively, and not all questions must be answered, how many different integers between -40 and 160 are not possible scores on this test?

(A) None; all are possible scores

(B) 1, 2, or 3

$\boxed{\text{(C)}}$ 4, 5, or 6

(D) 7, 8, or 9

(E) More than 9

$\boxed{\text{SOLN}}$ The impossible scores are 149, 153, 154, 157, 158, and 159. □

20. Call P_1 the probability of getting a sum of 7 when two fair dice are thrown. Call P_2 the probability of getting a sum of 7 when a fair die is thrown and a dial spun on a spinner with four equal sections numbered 1, 2, 3, and 4. Which statement is true?

$\boxed{\text{(A)}}$ $P_1 = P_2$

(B) $P_1 < P_2$

(C) $P_1 > P_2$

(D) $P_1 + P_2 > 1$

(E) There is not enough information.

$\boxed{\text{SOLN}}$ Both P_1 and P_2 are 1/6. Examine both sample spaces below. $P_1 = 6/36 = 1/6$. $P_2 = 4/24 = 1/6$.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

In probability $4/24 + 2/12 = 6/36$ (these are not fractions or rational numbers in the usual sense). □

21. What is the solution set of $|2x - 3| < \frac{1}{2}$?

(A) $\{x \,|\, x < \frac{5}{4}\} \cup \{x \,|\, x > \frac{7}{4}\}$

$\boxed{\text{(B)}}$ $\{x \,|\, x > \frac{5}{4}\} \cap \{x \,|\, x < \frac{7}{4}\}$

(C) $\{x \,|\, x \le \frac{5}{4}\} \cap \{x \,|\, x \ge \frac{7}{4}\}$

(D) $\{x \,|\, x > \frac{5}{4}\} \cup \{x \,|\, x < \frac{7}{4}\}$

(E) None of these

$\boxed{\text{SOLN}}$ $|2x - 3| < \frac{1}{2} \quad \Rightarrow$

$$\begin{array}{ccc} (2x-3) < \frac{1}{2} & \text{and} & -(2x-3) < \frac{1}{2} \\ 2x < \frac{7}{2} & \text{and} & -2x < -\frac{5}{2} \\ x < \frac{7}{4} & \text{and} & x > \frac{5}{4} \end{array}$$

□

22. What is the output of the following computer program?

```
10 for i = 1 to 3
20 for j = 1 to 3
30 if i < j then print i:
            else print j
40 next j
50 next i
```

$\boxed{\text{(A)}}$ 1, 1, 1, 1, 2, 2, 1, 2, 3

(B) 1, 2, 3, 2, 2, 3, 3, 3, 3

(C) 1, 2, 2, 3, 3, 3

(D) 1, 2, 3, 3, 2, 2, 2, 1, 1

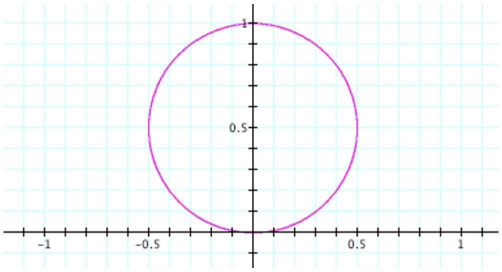
(E) 1, 2, 3, 1, 2, 3, 1, 2, 3

$\boxed{\text{SOLN}}$

i	j	i if $i < j$; else j
1	1	1
1	2	1
1	3	1
2	1	1
2	2	2
2	3	2
3	1	1
3	2	2
3	3	3

$\square$

23. Which polar equation best represents the graph?



(A) $r = \theta$

$\boxed{\text{(B)}}$ $r = \sin\theta$

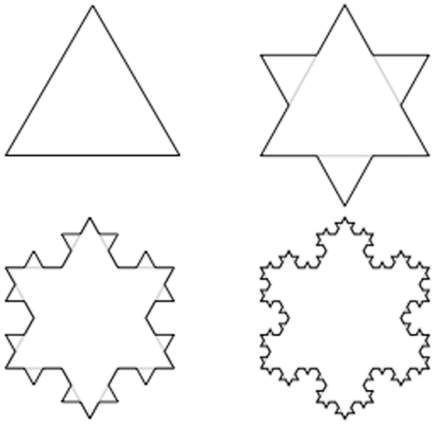
(C) $r = \cos\theta$

(D) $r = 1$

(E) $\theta = 2\pi$

$\boxed{\text{SOLN}}$ The point $(r, \theta) = (0, 0)$ eliminates choices D and E. $r = \theta$ is a spiral, so A is eliminated. Check other points to verify $r = \sin\theta$. $\square$

24. The Koch snowflake is constructed by starting with an equilateral triangle with sides of length 1. Then construct an equilateral triangle on the middle third of each side and erase the base of each of the new triangles. Repeat indefinitely.



The original perimeter is 3. After the first iteration the perimeter is 4. What is the perimeter after n iterations?

(A) $3n$

(B) $\frac{4}{3}n$

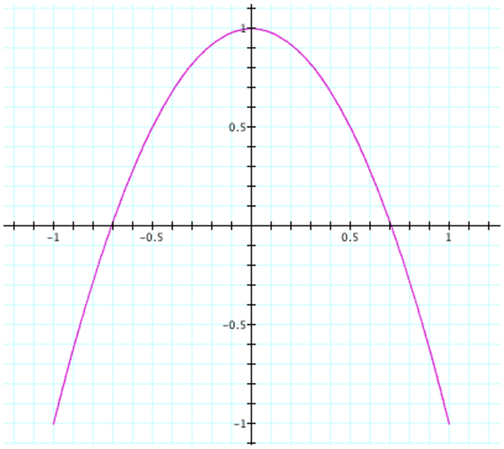
(C) $3(\frac{4}{3}n)$

$\boxed{\text{(D)}}$ $3(\frac{4}{3})^n$

(E) $(3n)^{4/3}$

$\boxed{\text{SOLN}}$ The Koch snowflake is a very interesting beast; its fractal dimension is $4/\log 3 \approx 1.26$, which is greater than the dimension of a line but less than the dimension of a 2D area. The perimeter after each iteration is $3, 3(\frac{4}{3}), 3(\frac{4}{3})^2, \ldots, 3(\frac{4}{3})^n$ which is a geometric sequence with a common ratio of $4/3$, so it is a divergent sequence. That is, the perimeter of the Koch snowflake is infinite! But the area enclosed by the snowflake is finite: $A = 2\sqrt{3}/5$. Google "snowflake sweep" to learn about fractal curves which fill the interior of the snowflake and have a dimension of 2. $\square$

25. Which set of parametric equations will produce the graph shown?



(A) $\begin{cases} x(t) = \sin t \\ y(t) = \cos t \end{cases} \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

(B) $\begin{cases} x(t) = t \\ y(t) = t^2 \end{cases} \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

(C) $\begin{cases} x(t) = t \\ y(t) = \cos t \end{cases} \quad -1 \le t \le 1$

(D) $\begin{cases} x(t) = \sin 2t \\ y(t) = \cos t \end{cases} \quad 0 \le t \le 2\pi$

$\boxed{\text{(E)}}$ $\begin{cases} x(t) = \sin t \\ y(t) = \cos 2t \end{cases} \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$\boxed{\text{SOLN}}$ Do trial and error by plugging in the beginning, middle, and ending values for t into the equations for x and y to find a few ordered pairs (x, y). Or plug $\sin t$ in for x and $\cos 2t$ in for y in $y = -2x^2 + 1$ to get an identity. □

26. Given that $\ln 1 = 0$, $\ln 5 = 1.6094$, and $\ln 2 = 0.6931$, find $\ln 0.2$.

(A) -0.06931

(B) 1.0644

$\boxed{\text{(C)}}$ -1.6094

(D) 0.06931

(E) None of these

$\boxed{\text{SOLN}}$ $\ln 0.2 = \ln \frac{1}{5} = \ln(5^{-1}) = -\ln 5$ □

27. Think of something so hot it glows, e.g., a branding iron. The power (energy/time) radiated by a blackbody per unit area is proportional to the fourth power of the absolute temperature.

$$\frac{P}{A} \propto T^4$$

By what factor is the power radiated from a given blackbody increased if its temperature is raised from 100 K to 300 K?

(A) 3

(B) 9

(C) 27

(D) 300

$\boxed{\text{(E)}}$ None of the above

$\boxed{\text{SOLN}}$ The answer is $\left(\frac{300\,\mathrm{K}}{100\,\mathrm{K}}\right)^4 = 3^4 = 81$. (The constant of proportionality is the Stefan-Boltzmann constant: $\sigma = 5.6704 \times 10^{-8}\,\mathrm{W/m^2 \cdot K}$.) The temperatures must be expressed on an absolute scale, such as in kelvins, for the formula to work. □

28. All numbers in this problem are in base five. What is $234 + 331$?

(A) 565

(B) 4302

$\boxed{\text{(C)}}$ 1120

(D) 160

(E) 3041

$\boxed{\text{SOLN}}$ Add vertically in base five.

$$\begin{array}{r} 234 \\ \underline{+331} \\ 1120 \end{array}$$

Or change everything to base ten. $234_5 = 69_{10}$ and $331_5 = 91_{10}$. Then $69_{10} + 91_{10} = 160_{10}$. Converting 160_{10} back to base five gives $(1 \times 125) + (1 \times 25) + (2 \times 5) + (0 \times 1) = 1120_5$. □

29. Ephraim has a plow that can clear 3 in of snow from the streets in 6 h. By using this plow and another plow from Manti at the same time, the same amount of snow can be cleared in 2.4 h. How long would it take the plow from Manti to do the job alone?

(A) 1.8 h

(B) 2.2 h

(C) 3.6 h

(D) 4.2 h

$\boxed{\text{(E)}}$ None of these

$\boxed{\text{SOLN}}$ In each row of the table rate × time = job.

	$r\ (\frac{\text{in}}{\text{h}})$	$t\,(\text{h})$	$J\,(\text{in})$
Ephraim	0.5	6	3
Manti	r_{M}	t_{M}	3
Both	$r_{\text{M}} + 0.5$	2.4	3

In the last column of the table, the job is to clear 3 in of snow in each case. In the rate column $r_{\text{both}} = r_{\text{E}} + r_{\text{M}}$. Our goal is to find t_{M}, which, from the Manti row, is $t_{\text{M}} = 3/r_{\text{M}}$. So we'll first use the last row to find r_{M}.

$$\left(r_{\text{M}} + 0.5\,\frac{\text{in}}{\text{h}}\right) \cdot 2.4\,\text{h} = 3\,\text{in}$$

$$r_{\text{M}} = \frac{3\,\text{in}}{2.4\,\text{h}} - 0.5\,\frac{\text{in}}{\text{h}} = 0.75\,\frac{\text{in}}{\text{h}}$$

$$t_{\text{M}} = \frac{3\,\text{in}}{0.75\,\frac{\text{in}}{\text{h}}} = 4\,\text{h}$$

$\square$

30. A cube has a volume of $512\,\text{cm}^3$.
What is the surface area of the cube?

$\boxed{\text{(A)}}$ $384\,\text{cm}^2$

(B) $256\,\text{cm}^2$

(C) $216\,\text{cm}^2$

(D) $96\,\text{cm}^2$

(E) None of these

$\boxed{\text{SOLN}}$ For a cube all edges are equal.

$$V = L \times L \times L \quad \Rightarrow \quad L = \sqrt[3]{V} = 8\,\text{cm}$$

Each side has area $L \times L = 64\,\text{cm}^2$. There are six sides for a total surface area of $6 \times 64\,\text{cm}^2 = 384\,\text{cm}^2$. $\square$

31. Upon investigating, the police found that one of a group of four girls had stolen a car. The police knew that three of the girls would always tell the truth, but that one consistently lied. When questioned, the girls made the following statements:

Alice: Betty did it.
Betty: Deb did it.
Carla: I didn't do it.
Deb: Betty lied when she said I did it.

Who stole the car?

(A) Alice

$\boxed{\text{(B)}}$ Betty

(C) Carla

(D) Deb

(E) Not enough information

$\boxed{\text{SOLN}}$ Alice and Betty can't both be telling the truth, so one of them is the liar. If Alice is the liar then Betty is telling the truth; but if there is only one liar and it is Alice, then Deb's claim that Betty also lied can't be right. Therefore, Betty must be the liar (which squares with the truth told by Deb), so Alice is telling the truth that Betty did it. $\square$

32. The average of A and $2B$ is 7, and the average of A and $2C$ is 8. What is the average of A, B, and C?

(A) 3

(B) 4

$\boxed{\text{(C)}}$ 5

(D) 6

(E) 9

$\boxed{\text{SOLN}}$ The info given is

$$\frac{A+2B}{2}=7 \qquad \frac{A+2C}{2}=8$$

Add the two equations together.

$$\frac{2A+2B+2C}{2}=15$$

Cancel the 2's. Then $A+B+C=15$ so the average is 5. $\square$

33. Find the determinant: $\begin{vmatrix} \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & \mathrm{i} \end{vmatrix}$

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\boxed{\text{(B)}}$ 0

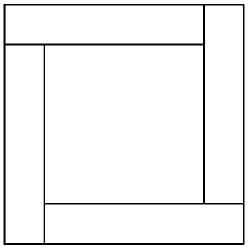
(C) 1

(D) 2

(E) None of the above

$\boxed{\text{SOLN}}$ $(\mathrm{i})(\mathrm{i})-(-\mathrm{i})(-\mathrm{i})=-1+1=0$ $\square$

34. A square is covered by a design made up of four identical rectangles surrounding a central square, as shown. If the area of the central square is 4/9 the area of the entire design, find the ratio of the length of a rectangle to the side of the central square.



$\boxed{\text{(A)}}$ 5/4

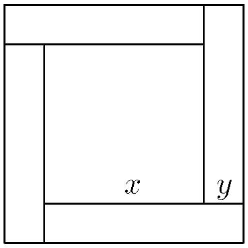
(B) 4/3

(C) 7/5

(D) 3/2

(E) 8/5

$\boxed{\text{SOLN}}$ Call the side of the central square x and the width of a rectangle y. What we want is $\frac{x+y}{x}$.



From what we are given we simplify.

$$\begin{aligned} x^2 &= \frac{4}{9}(x+2y)^2 \\ x &= \frac{2}{3}(x+2y) \\ x &= \frac{2}{3}x+\frac{4}{3}y \\ \frac{1}{3}x &= \frac{4}{3}y \\ x &= 4y \end{aligned}$$

Now plug this into what we want.

$$\frac{x+y}{x}=\frac{4y+y}{4y}=\frac{5}{4}$$

$\square$

35. Matrix multiplication is not commutative in general. However, the following matrices A and B do commute. What is their product?

$$A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \quad B = \begin{bmatrix} \cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha-\beta) \\ \sin(\alpha-\beta) & \cos(\alpha+\beta) \end{bmatrix}$

(C) $\begin{bmatrix} \cos(\alpha-\beta) & \sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha-\beta) \end{bmatrix}$

$\boxed{\text{(D)}}$ $\begin{bmatrix} \sin(\alpha+\beta) & \cos(\alpha-\beta) \\ \cos(\alpha-\beta) & \sin(\alpha+\beta) \end{bmatrix}$

(E) $\begin{bmatrix} \sin(\alpha-\beta) & \cos(\alpha+\beta) \\ \cos(\alpha+\beta) & \sin(\alpha-\beta) \end{bmatrix}$

$\boxed{\text{SOLN}}$ The first element of the product is $\sin\alpha\cos\beta + \cos\alpha\sin\beta$. This is a trig identity which equals $\sin(\alpha+\beta)$. This is enough to eliminate all incorrect choices.
The product of commuting symmetric matrices is symmetric because $(AB)^T = B^T A^T$. □

36. Which equation best represents the graph?

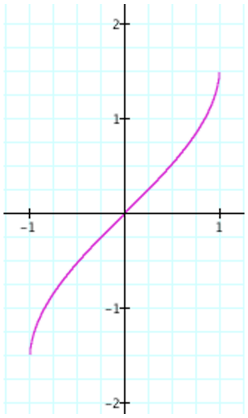
(A) $y = \tan^{-1} x$

(B) $y = \cos^{-1} x$

(C) $y = \sec^{-1} x$

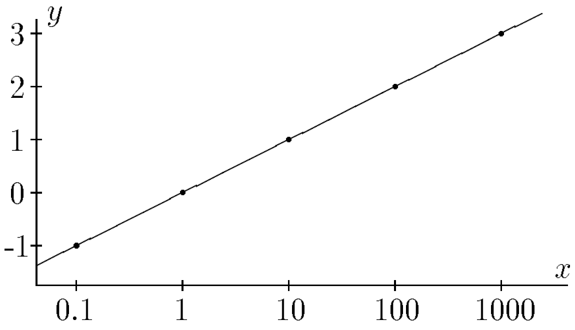
(D) $y = \csc^{-1} x$

$\boxed{\text{(E)}}$ $y = \sin^{-1} x$



$\boxed{\text{SOLN}}$ $y = \sin^{-1} x$ is the only one that gives both points $(0,0)$ and $(1, \frac{\pi}{2})$. □

37. Which equation best represents the graph?



(A) $y = e^x$

(B) $y = mx + b$

(C) $y = 10^x$

$\boxed{\text{(D)}}$ $y = \log x$

(E) $y = \frac{1}{2}x$

$\boxed{\text{SOLN}}$ This is a semilog plot called a lin-log plot because the y-axis is a linear scale and the x-axis is a logarithmic scale. A straight line on a lin-log plot is a logarithmic function. □

38. It's Sophie's birthday! Sophie Germain, famous, self-taught, woman mathematician, was born April 1, 1776. A prime number p is called a *Sophie Germain prime* if $2p+1$ is also prime. All Sophie Germain primes $p > 3$ are equivalent to ______ (mod 6).

(A) 1

(B) 2

(C) 3

(D) 4

$\boxed{\text{(E)}}$ 5

$\boxed{\text{SOLN}}$ Take any Sophie Germain prime > 3, such as 5, 11, 23, 29, etc. mod 6, and you get 5. The matching safe prime $2p + 1$ is also equivalent to 5 (mod 6). □

39. Add any integer n to the square of $2n$ to produce an integer m. For how many values of n is m prime?

(A) 0

(B) 1

$\boxed{\text{(C)}}$ 2

(D) a finite number > 2

(E) ∞

$\boxed{\text{SOLN}}$ $(2n)^2 + n = m$ is divisible by n, so it (i.e., m) can't be prime unless what we divided by was 1 (or -1). Check that $n = 1$ and $n = -1$ produce prime m's. But no other integers do. $\square$

40. A door is 4 ft wide and 7 ft tall. If the door is standing open at a 90° angle with the door frame, what is the greatest distance in feet from the outer top corner of the door to a point on the door frame?

(A) 8

$\boxed{\text{(B)}}$ 9

(C) 9.5

(D) 10

(E) 11

$\boxed{\text{SOLN}}$ Think three dimensionally. The farthest point is 4 ft in one direction, 4 ft in the second direction, and 7 ft in the third. $\sqrt{4^2 + 4^2 + 7^2} = 9$ $\square$