Snow College Jr. Mathematics Contest

key

March 19, 2024 **Junior Division: Grades 7–9** **Form: T**

Bubble in clearly the single best choice for each question you choose to answer.

1. What is the correct time exactly 540 seconds after midnight?

   (A) 12:09 p.m.

   (B) 12:09 a.m.

   (C) 9 p.m.

   (D) 9 a.m.

   (E) 12:54 a.m.

   SOLN $\dfrac{540\,\text{s}}{60\,\text{s/min}} = 9\,\text{min}$ □

2. The sum of four consecutive even integers is 148. What is the **sum** of the digits of the smallest of the four?

   (A) 6

   (B) $7 = 3 + 4$

   (C) 9

   (D) 12

   (E) 14

   SOLN $n + (n+2) + (n+4) + (n+6) = 148$
   $4n + 12 = 148 \Rightarrow 4n = 136 \Rightarrow n = 34$ □

3. Kim plays basketball for her school. Her free-throw shooting percentage for the season was 75% exactly before today. During tonight's game she makes all five free throws, bringing her percentage up to 80%. How many free throws has Kim made on the season (including tonight)?

   (A) 20

   (B) 22

   (C) 24

   (D) 25

   (E) 28

   SOLN Call her shots taken before tonight x. Her shots made can be represented as

   $$\frac{3}{4}x + 5 = \frac{4}{5}(x+5)$$

   where the left side is shots made before tonight + 5, and the right side is 80% of all shots made. Solving for x gives $x = 20$; then the left side becomes $15 + 5 = 20$. □

4. Towns A, B, and C are at the corners of a triangle with equal sides. A car travels at constant speeds from A to B at 30 mph, from B to C at 40 mph, and from C back to A at 60 mph. What is the average speed for the round trip?

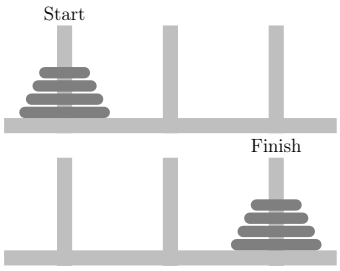
(A) 40 mph
(B) 43 mph
(C) 45 mph
(D) 48 mph
(E) 50 mph

SOLN The answer is same for any equilateral triangle, but let's assume a specific case for ease of illustration: $s = 120$ mi, so the total trip is 360 mi. The first leg takes (120 mi)/(30 mi/h) = 4 h; likewise, the second leg takes 3 h, and the last leg takes 2 h, for a total of 9 h. The average speed is 360 mi/9 h = 40 mi/h.

Marilyn vos Savant in Parade, Dec. 17, 2017. □

5. Four rings of different sizes are stacked on one of three posts in ascending order (smallest on top). You are able to move one ring at a time (taking the top ring from one post and moving it to another post), but you may never place a larger ring on a smaller ring. What is the minimum number of moves required to move the entire stack to a different post?

(A) 12
(B) 14
(C) 15
(D) 16
(E) 17



SOLN This is the famous Towers of Hanoi game. Look for a pattern. To move one ring to a different post requires one move. Two rings require three moves, etc. n rings require $2^n - 1$ moves. □

6. The product of the lengths of the diagonals of a square is 72. What is the length of the sides of the square?

(A) 4
(B) 5
(C) 6
(D) 8
(E) 9

SOLN The diagonals have equal length, so $d^2 = 72$. If x is the length of a side, the Pythagorean Thm gives $2x^2 = d^2 = 72$, so $x = 6$. □

7. The shadow cast by a tall tree is 6 m long. At the same time of day and at the same location, an upright meter stick casts a shadow of 20 cm. How tall is the tree?

(A) 20 m
(B) 24 m
(C) 27 m
(D) 28 m
(E) 30 m

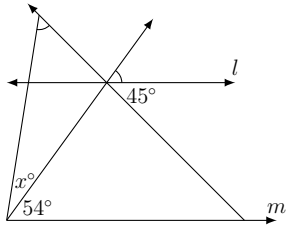
SOLN Similar triangles give proportional measurements, so $x/6 = 1/0.20$ giving $x = 30$. □

8. The sum of the first n counting numbers is 210: $1 + 2 + 3 + \cdots + n = 210$. Find n.

(A) 14
(B) 16
(C) 17
(D) 20
(E) 24

SOLN $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. $210 = \frac{n(n+1)}{2}$ $420 = n^2 + n \implies 0 = n^2 + n - 420 \implies n = 20$. Then $\frac{20(21)}{2} = 210$. □

9. In the following diagram, lines l and m are parallel. Find the measure of angle x.



(A) $21°$
(B) $25°$
(C) $27°$
(D) $45°$
(E) $54°$

SOLN Consider the small triangle in the upper left. The bottom left angle in this triangle is $x + 54°$ as this corresponds to the bottom left angle sum. Because corresponding angles of parallel lines are congruent, the two angles marked congruent are also $54°$. The third angle is $45°$ as this is opposite the other marked angle. Thus, $x° + 54° + 54° + 45° = 180°$ or $x° = 27°$. □

10. Compute the following sum in base 2.

$$\begin{array}{r} 1101101 \\ +\ 111011 \\ \hline \end{array}$$

(A) 10001100
(B) 11010111
(C) 10110010
(D) 11011101
(E) 10101000

SOLN In base 2, $1 + 0 = 0 + 1 = 1$ and $1 + 1 = 10$ where the 1 is carried to the next place value. □

11. For the function $f(x) = x^2 + 2x - 5$, compute the value of $f(f(f(1)))$.

(A) -5
(B) 5
(C) 10
(D) 12
(E) 115

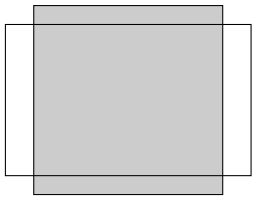
SOLN $f(1) = 1^2 + 2(1) - 5 = -2$ and $f(-2) = (-2)^2 + 2(-2) - 5 = -5$ and $f(-5) = (-5)^2 + 2(-5) - 5 = 10$. □

12. One cubic centimeter is equal to how many cubic millimeters?

(A) 10
(B) 100
(C) 1000
(D) 10 000
(E) 1 000 000

SOLN There are 10 mm per cm, so 10 mm $\times$ 10 mm $\times$ 10 mm $= 1000\,\text{mm}^3$. □

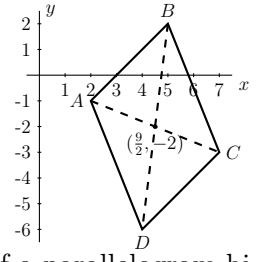
13. One side of the gray square is increased by 3 cm while its adjacent side is decreased by 2 cm. The perimeter of the resulting rectangle is 22 cm. What is the area of the original gray square?



(A) $9\,\text{cm}^2$
(B) $16\,\text{cm}^2$
(C) $25\,\text{cm}^2$
(D) $64\,\text{cm}^2$
(E) $121\,\text{cm}^2$

SOLN If x is a side length of the square, the perimeter of the new rectangle will be $22 = 2(x + 3) + 2(x - 2) = 4x + 2$. Solving gives $x = 5$ and the square's area is $25\,\text{cm}^2$. □

14. Find the intersection point of the diagonals of the parallelogram $ABCD$ for $A(2, -1)$, $B(5, 2)$, $C(7, -3)$, and $D(4, -6)$.



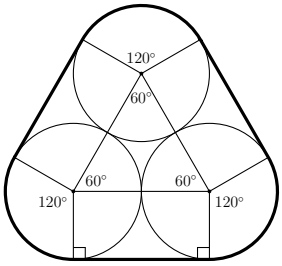
(A) $(\frac{9}{2}, -2)$
(B) $(4, -2)$
(C) $(5, -3)$
(D) $(\frac{9}{2}, -3)$
(E) $(\frac{9}{2}, -\frac{5}{2})$

SOLN The diagonals of a parallelogram bisect each other. Find the midpoint between opposite vertices.

$$\left(\frac{2+7}{2}, \frac{-1-3}{2}\right)$$

□

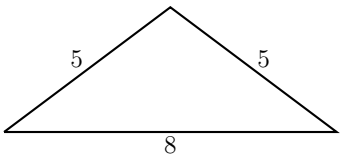
15. Three disks of radius 1 cm are mutually tangent as in the figure below. A rubber band is wrapped around the outside of the group. Find the total length of the band in cm.



(A) $3+\pi$
(B) 3π
(C) $3+2\pi$
(D) 6π
[(E)] $6+2\pi$

SOLN Connecting the centers of the circle forms an equilateral triangle with side lengths of 2 cm. Connecting the centers to the points of tangency shows that the portion of each disk where the band touches has a central angle of 120°. The band covers three of these sections or 360° which has a circumference of 2π cm. Combine this with the three segments of length 2 cm from the rectangles. □

16. What is the area of the triangle?



[(A)] 12
(B) 12.5
(C) 20
(D) 25
(E) 40

SOLN The short way is Heron's formula: $A=\sqrt{s(s-a)(s-b)(s-c)}$, where s is half of the perimeter and $a, b,$ & c are the side lengths.

$$\sqrt{9(9-5)(9-5)(9-8)}=$$

$$=\sqrt{(9)(4)(4)(1)}=(3)(4)=12$$

Alternatively, drop a vertical from the top creating two 3-4-5 right triangles, each of which has $B=4$, $H=3$. □

17. The number 6545 can be written as a product of a pair of positive two-digit integers. What is the sum of the two integers?

(A) 156
[(B)] 162
(C) 187
(D) 238
(E) 166

SOLN The prime factorization of 6545 is $5\cdot 7\cdot 11\cdot 17$. The only way to combine them into a pair of products (each a two-digit number) is 77 and 85. □

18. Which whole number is closest to the ratio?

$$\frac{10^{2023}+10^{2025}}{10^{2024}+10^{2024}}$$

(A) 1
(B) 2
(C) 4
[(D)] 5
(E) 10

SOLN Factoring out 10^{2023} on the top and bottom gives

$$\frac{1+10^2}{10^1+10^1}=\frac{101}{20}$$

□

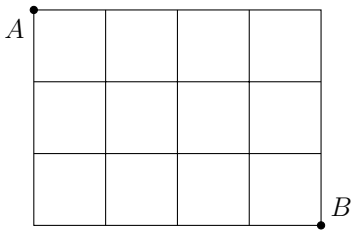
19. Find the median: 2, 5, 10, 8, 2, 4, 9, 9, 7, 9.

(A) 7
[(B)] 7.5
(C) 10
(D) 9
(E) 6.5

SOLN The median of an odd number of numbers is the middle number (once put in order). The median of an even number of numbers is the average of the middle two (after ordering). □

20. Going only right or down, how many different ways are there to get from point A (upper left corner) to point B (lower right corner) of the 3×4 grid below?

(A) 28
(B) 32
(C) 35
(D) 56
(E) 84



SOLN The trip will take exactly 7 steps, 3 of which are down and four are to the right. Since order doesn't matter, this is a combination: ${}_7C_3 \equiv \binom{7}{3} = {}_7C_4 \equiv \binom{7}{4} = 35$. One can also note the entries of Pascal's triangle at each corner. □