Snow College Jr. Mathematics Contest

key

April 4, 2023 **Junior Division: Grades 7–9** **Form: T**

Bubble in clearly the single best choice for each question you choose to answer.

1. What is the greatest positive integer that evenly divides the sum of any five consecutive positive integers?

   (A) 2
   (B) 3
   (C) 4
   $\boxed{\text{(D)}}$ 5
   (E) 6

   SOLN The sum of any five consecutive whole numbers can be written $n + (n+1) + (n+2) + (n+3) + (n+4) = 5n + 10 = 5(n+2)$, so 5 must divide the sum. Even easier: label the middle number n. Then $(n-2)+(n-2)+n+(n+1)+(n+2) = 5n$. □

2. Finish the square so that the numbers in each row, column, and diagonal add up to the same number. What is the sum of the missing numbers?

   $\boxed{\text{(A)}}$ 57
   (B) 59
   (C) 62
   (D) 66
   (E) 69

15		
	11	
9		**7**

SOLN

15	5	**13**
9	**11**	13
9	17	**7**

$5 + 13 + 9 + 13 + 17 = 57$

A second solution does not require knowing the missing numbers: since all rows, columns, and diagonals sum to 33, then 3×33 is the total of all numbers. Subtract 33 for the complete diagonal and 9 for the corner. $3 \times 33 - 33 - 9 = 57$ □

3. If you add the measure of any 2 angles of triangle T, the sum is always $120°$. Triangle T **must** be

   $\boxed{\text{(A)}}$ equiangular.
   (B) scalene.
   (C) right.
   (D) obtuse.
   (E) geodesic.

   SOLN The third angle will always be $60°$. (BTW, a geodesic triangle is one drawn on a curved surface; for example, a geodesic triangle on the surface of the earth could have three right angles.) □

4. What is the sum of all odd integers between 51 and 375 inclusive?

   (A) 4260
   (B) 5792
   $\boxed{\text{(C)}}$ 34719
   (D) 43876
   (E) 51892

   SOLN We have an arithmetic sequence with $a_1 = 51$, $d = 2$. The formula for the nth term is $a_n = a_1 + d(n-1)$. When $a_n = 375$, $n = 163$. The sum is $s_n = (a_1 + a_n)(\frac{n}{2}) = (51 + 375)(\frac{163}{2})$. □

5. The city of Königsberg has 7 bridges. Suppose we wanted to take a walk through the city eventually crossing each bridge. What is the minimum number of bridge crossings required to cross every bridge in the city?

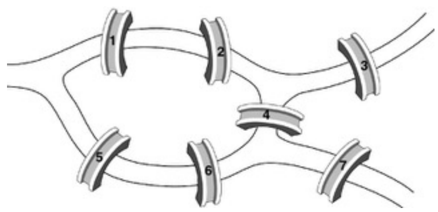
(A) 7

$\boxed{\text{(B)}}$ 8

(C) 9

(D) 10

(E) 11



$\boxed{SOLN}$ In 1736 Leonhard Euler proved using graph theory that there could be no fewer than 8 crossings on this map. □

6. How many of the 15 positive factors of 400 are evenly divisible by 4?

(A) 4

(B) 8

$\boxed{\text{(C)}}$ 9

(D) 10

(E) 11

$\boxed{SOLN}$ Find all factor pairs and check for divisibility by 4: $1 \cdot 400, 2 \cdot 200, 4 \cdot 100, 5 \cdot 80, 8 \cdot 50, 10 \cdot 40, 16 \cdot 25, 20 \cdot 20$. The factors that are divisible by 4 are 400, 200, 100, 80, 40, 20, 16, 8, 4. □

7. Your French class is randomly choosing two students for an all-expense-paid trip to Paris. If your class has 20 students, what is the probability that you will be chosen?

(A) $\frac{1}{9}$

$\boxed{\text{(B)}}$ $\frac{1}{10}$

(C) $\frac{1}{19}$

(D) $\frac{1}{20}$

(E) $\frac{1}{190}$

$\boxed{SOLN}$ The number of 2-person groups that include you is 19 (you matched with each of your classmates). The total number of 2-person groups is ${}_{20}C_2 = \binom{20}{2} = \frac{20!}{18! \cdot 2!} = 190$. So the probability is $\frac{19}{190}$. Shorter: You not chosen $= \frac{19}{20} \cdot \frac{18}{19} = \frac{18}{20}$. So you chosen is $1 - \frac{9}{10}$. Shortest: 2 chosen out of 20 total $= \frac{2}{20}$. □

8. There are several kinds of averages or means. One of them is the geometric mean, which is used often in computing growth rates. The geometric mean of x and y is $\sqrt{xy}$. Compute the geometric mean of 18 and 50.

(A) 20

$\boxed{\text{(B)}}$ 30

(C) 34

(D) $\frac{225}{17}$

(E) 16

$\boxed{SOLN}$ $\sqrt{18 \cdot 50} = \sqrt{2^2\, 3^2\, 5^2} = \sqrt{900} = 30$ □

9. My coin jar has 100 pennies, 200 nickels, 300 dimes, and 400 quarters in it. What is the total value of the coins?

(A) \$91

(B) \$121

$\boxed{\text{(C)}}$ \$141

(D) \$161

(E) \$191

$\boxed{SOLN}$ 100 pennies = \$1; 200 nickels = \$10; 300 dimes = \$30; 400 quarters = \$100. □

10. A square of side-length 4π has the same perimeter as a circle of what diameter?

(A) 2

(B) 4

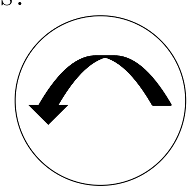
(C) 8

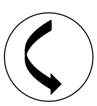
(D) 12

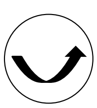
$\boxed{\text{(E)}}$ 16

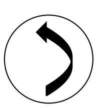
SOLN The perimeter of the square is $4 \cdot 4\pi$ and the perimeter of the circle is πD. □

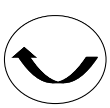
11. For the following figure, which choice shows a $90°$ clockwise rotation followed by a reflection across a vertical axis?

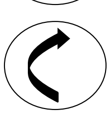


(A) 

(B) 

(C) 

(D) 

$\boxed{\text{(E)}}$ 

SOLN The rotation produces choice C and then the reflection produces choice E. □

12. A basketball player has a free throw shooting average of 83%. She is fouled on a 3-point attempt. What is the probability that she will make all three free throws?

(A) 42.3%

$\boxed{\text{(B)}}$ 57.2%

(C) 83%

(D) 49%

(E) 27.7%

SOLN $(0.83)^3 = 0.572$
A quick approach is to approximate 0.83 by 0.8; this is easy to cube as 0.512 which is greater than 0.50. The answer must be less than 83% and the only other choice greater than 0.50 is the correct one. □

13. It can be shown that the sum of the squares of the first k natural numbers, $\sum_{j=1}^{k} j^2$, has a value of $k(k+1)(2k+1)/6$. Compute the sum of squares of the first 15 natural numbers.

$\boxed{\text{(A)}}$ 1240

(B) 1200

(C) 930

(D) 1440

(E) 1015

SOLN
$$\frac{15 \times 16 \times 31}{6} = 1240$$
□

14. If you place a cake of soap on a pan of a scale and $\frac{3}{4}$ cake of soap and a $\frac{3}{4}$-kg weight on the other, the pans balance. How much does a cake of soap weigh?

$\boxed{\text{(A)}}$ 3 kg

(B) 1 kg

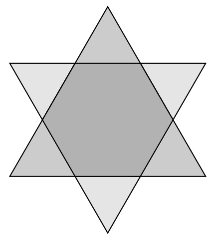
(C) $\frac{3}{4}$ kg

(D) $\frac{1}{2}$ kg

(E) $\frac{1}{4}$ kg

SOLN Since $\frac{1}{4}$ cake weighs $\frac{3}{4}$ kg, an entire cake weighs 3 kg.
$$x = \tfrac{3}{4}x + \tfrac{3}{4} \implies x = 3$$
□

15. Two congruent equilateral triangles each with area of $14\,\text{cm}^2$ overlap to form a regular hexagon as shown below. How many square centimeters is each of the small exterior triangles?



(A) 9/15

(B) 14/15

$\boxed{\text{(C)}}$ 14/9

(D) 15/9

(E) 14/6

$\boxed{\text{SOLN}}$ Fold the triangles outside of the hexagon back in and you cover the hexagon. One triangle is $14\,\text{cm}^2$ which can be broken into 9 smaller triangles equal to the exterior in size. Each one will have an area of $14/9\,\text{cm}^2$. □

16. Let $A = \{\text{perfect squares} < 100\}$ and $B = \{\text{multiples of } 3\}$. How many natural numbers are in $A \bigcap B$?

(A) 0

(B) 1

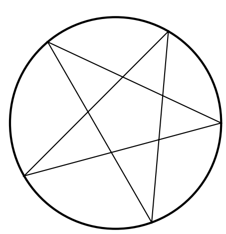
(C) 2

$\boxed{\text{(D)}}$ 3

(E) 4

$\boxed{\text{SOLN}}$ Perfect squares less than 100 that are also multiples of 3 are 9, 36, and 81. □

17. A circle circumscribes a five-pointed star. What is the sum of the five interior angles of the star?



(A) 90°

$\boxed{\text{(B)}}$ 180°

(C) 300°

(D) 360°

(E) Not enough information

$\boxed{\text{SOLN}}$ Each inscribed angle is half the measure of the intercepted arc. The five angles together intercept the entire circle so half this measure is 180°. □

18. What is the 10s digit of the smallest 3 digit palindrome (same forwards and backwards) whose digits add to 18?

(A) 4

(B) 5

(C) 6

(D) 7

$\boxed{\text{(E)}}$ 8

$\boxed{\text{SOLN}}$ The smallest 3-digit palindrome is 101. To add to 18, we need a minimum of 10 as the sum of the first and last digit which would be 5. The middle digit would then be 8 to get 585. □

19. Simplify the expression $\sqrt{\sqrt[0.06]{3^{0.12}}}$

(A) 2

(B) $2\sqrt{3}$

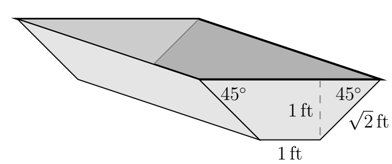
(C) $3\sqrt{2}$

$\boxed{\text{(D)}}$ 3

(E) $6\sqrt{2}$

$\boxed{\text{SOLN}}$ Radicals as rational exponents gives $((3^{0.12})^{1/0.06})^{1/2} = (3^2)^{1/2}$. □

20. The perimeter of the rectangular top rim of the vat shown below is 26 ft. How many cubic feet of water will the vat hold if it is 1 ft deep?



(A) $12\sqrt{2}$

(B) $14\sqrt{2}$

$\boxed{\text{(C)}}$ 20

(D) 22

(E) 26

$\boxed{\text{SOLN}}$ With the angles of 45°, the top of the trapezoid face will be 3 ft. The perimeter is $26 = 2(3) + 2y$, so the length of the vat is 10 ft. The trapezoid face has an area of $\frac{1}{2}(1)(1+3) = 2$ so the total volume is $(10)(2) = 20\,\text{ft}^3$. □