Snow College Jr. Mathematics Contest

$\boxed{\text{key}}$

April 2, 2019 **Junior Division: Grades 7–9** **Form:** $\boxed{\mathsf{T}}$

Bubble in the single best choice for each question you choose to answer.

1. The time it takes for one cycle of a repetitive motion is called the period. The period of the earth's orbit around the sun is 1 year. What is the period of the vibrating tines of a tuning fork whose frequency is 1000 cycles per second?

   (A) 0.00001 s
   $\boxed{\text{(B)}}$ 0.001 s
   (C) 0.1 s
   (D) 10 s
   (E) 1000 s

   $\boxed{\mathcal{SOLN}}$ The period T is the inverse of the frequency f. $1/1000 = 0.001$ □

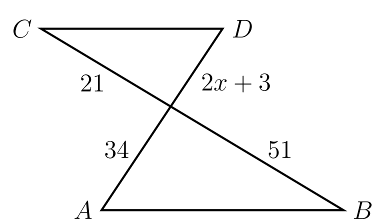
2. How many two-digit whole numbers are exactly 7 times the sum of their digits?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   $\boxed{\text{(E)}}$ 4

   $\boxed{\mathcal{SOLN}}$ $10a+b = 7(a+b) \implies 3a = 6b \implies a = 2b \qquad \{21, 42, 63, 84\}$ □

3. If line segments $\overline{AB}$ and $\overline{CD}$ are parallel, find the value of x.

   (A) 31.5
   (B) 0.5
   (C) 17.5
   (D) 15.5
   $\boxed{\text{(E)}}$ 5.5



   $\boxed{\mathcal{SOLN}}$ $\dfrac{2x+3}{21} = \dfrac{34}{51} = \dfrac{2}{3} \implies 2x + 3 = 14$ □

4. A student had the following scores in math: 92, 100, 88, 77, 99, 85, 96. What is the difference between the median and the mean of these scores?

   $\boxed{\text{(A)}}$ 1
   (B) 3
   (C) 14
   (D) 23
   (E) 27

   $\boxed{\mathcal{SOLN}}$ Mean $=$ sum$/7 = 91$. Median is the center score $= 92$. $92 - 91 = 1$ □

5. Which number is the largest?

   (A) 1.600
   (B) 1.06
   (C) 1.660
   (D) 1.066
   $\boxed{\text{(E)}}$ $1.\bar{6}$

   $\boxed{\mathcal{SOLN}}$ $1.\bar{6} = 1.66666666...$ □

6. Kim ate three-eighths of the pizza, Gigi ate one-fourth, Moe ate one-third, and Socks ate the rest. What fraction did Socks eat?

   $\boxed{\text{(A)}}$ $\frac{1}{24}$
   (B) $\frac{1}{12}$
   (C) $\frac{1}{6}$
   (D) $\frac{1}{4}$
   (E) $\frac{1}{3}$

   $\boxed{\mathcal{SOLN}}$ $1 - \frac{3}{8} - \frac{1}{4} - \frac{1}{3} = \frac{24}{24} - \frac{9}{24} - \frac{6}{24} - \frac{8}{24}$ □

7. A semiprime is the product of two (not necessarily distinct) primes. They are very useful in cryptology because it is easy to multiply two primes together, but hard to factor a large semiprime. What is the sum of the semiprimes less than 20?

(A) 45

(B) 49

(C) 54

(D) 57

$\boxed{\text{(E)}}$ 58

$\boxed{\text{SOLN}}$ $4+6+9+10+14+15=58$ □

8. Shannon takes her favorite number, adds 5 to it, multiplies the answer by 10, subtracts 20 from the result and then drops the final 0. If Shannon's (correct) answer is 9, what is her favorite number?

(A) 5

$\boxed{\text{(B)}}$ 6

(C) 7

(D) 8

(E) 9

$\boxed{\text{SOLN}}$ $9=\frac{10(x+5)-20}{10}=x+3 \implies x=6$ □

9. A triangle with integer side lengths and positive area has no two sides equal. Find its least possible perimeter.

(A) 6

(B) 7

(C) 8

$\boxed{\text{(D)}}$ 9

(E) 10

$\boxed{\text{SOLN}}$ Each side must be less than the sum of the other two; thus $1+2+3$ is not possible. The next possibility that works: $2+3+4=9$. □

10. Circle with center O has radius 3, $AB=8$, and $\overline{AB}$ is tangent to the circle at B. If $\overline{BC}$ is a diameter of the circle, find CD.

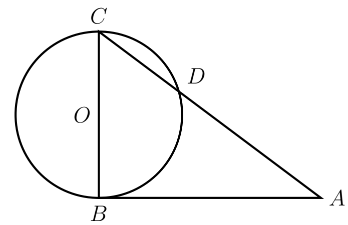
(A) 3

(B) 3.2

$\boxed{\text{(C)}}$ 3.6

(D) 4

(E) 6.4



$\boxed{\text{SOLN}}$ ΔABC is a right triangle with side lengths 8, 6, and 10 respectively. $\Delta BDC \sim \Delta ABC$ so $\frac{CD}{6}=\frac{6}{10}$. □

11. A set is closed under addition if the sum of any two elements is also in the set. Determine which of the five sets below is NOT closed under addition.

(A) $\{0\}$

(B) $\{2, 4, 6, 8, 10, \ldots\}$

(C) $\{15, 16, 17, 18, 19, \ldots\}$

$\boxed{\text{(D)}}$ $\{1, 3, 5, 7, 9, \ldots\}$

(E) $\{5, 10, 15, 20, \ldots\}$

$\boxed{\text{SOLN}}$ Example: $1+3=4$ is not in the set. □

12. $300 \div 200 = 1 \div$ _______.

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

$\boxed{\text{(C)}}$ $\frac{2}{3}$

(D) $\frac{3}{2}$

(E) $\frac{300}{200}$

$\boxed{\text{SOLN}}$

$$\frac{300}{200}=\frac{1}{x} \implies x=\frac{200}{300}$$

□

13. At *Wayside School*, Mrs. Jewls wanted to teach her students the difference between true and false. On her exam, the following two question appeared with instructions to answer as either **true** or **false**.

1. The answer to this statement is the same as the answer to statement number 2.

2. The answer to this statement is different from the answer to statement number 1.

Which of the following represents the correct answers to these two questions?

(A) 1. True – 2. True

(B) 1. True – 2. False

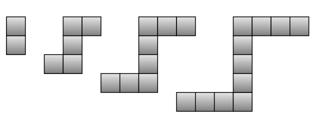
$\boxed{\text{(C)}}$ 1. False – 2. True

(D) 1. False – 2. False

(E) Impossible to tell.

$\boxed{\text{SOLN}}$ If (1) is true, then (2) cannot be true or false. So (1) must be false and then statement (2) will be true. □

14. Assume the following pattern of square tiles continues. Which term, if any, has exactly 449 tiles?



(A) 229^{th} term

(B) 221^{st} term

(C) 190^{th} term

$\boxed{\text{(D)}}$ 150^{th} term

(E) No term has exactly 449 tiles.

$\boxed{\text{SOLN}}$ The number of tiles across the top (or bottom) of the figure is the same as the number of the term; that is, the nth term has n tiles on the top. The middle tiles not already counted are $n-1$. The total is $2n+(n-1)=3n-1$. Setting $3n-1=449$ yields $n=150$. □

15. $\sqrt{16^{16}} =$

(A) 4^4

(B) 4^8

(C) 16^2

(D) 16^4

$\boxed{\text{(E)}}$ 16^8

$\boxed{\text{SOLN}}$ $\sqrt{16^{16}} = 16^{16/2} = 16^8$ □

16. If $x\nabla y$ is defined as $(x+1)(y-1)$, what is $4\nabla 3$?

(A) 3

(B) 5

(C) 8

$\boxed{\text{(D)}}$ 10

(E) 12

$\boxed{\text{SOLN}}$ $4\nabla 3 = (4+1)(3-1) = 5\cdot 2 = 10$ □

17. In astronomy, the apparent brightness b of a star is related to the luminosity L by

$$b = \frac{L}{4\pi d^2}$$

where d is the distance to the star. If planet 1 is five times farther from a star than planet 2 is, what will be the ratio b_1/b_2?

$\boxed{\text{(A)}}$ $\frac{1}{25}$

(B) $\frac{1}{5}$

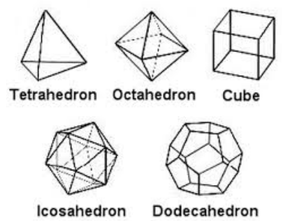
(C) 1

(D) 5

(E) 25

$\boxed{\text{SOLN}}$ This is called an inverse-square law. d is in the denominator and it is squared, so double the distance means one quarter the brightness. $\left(\frac{1}{5^2}\right) = \frac{1}{25}$ □

18. A die is *fair* if all faces are congruent and each number has an equal chance of being rolled. Which is not a possible number of faces for a fair die?

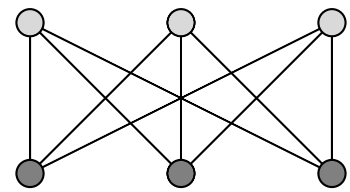


`https://tinyurl.com/y49yu9q2`

- (A) 4
- $\boxed{\text{(B)}}$ 5
- (C) 8
- (D) 12
- (E) 20

SOLN All of the Platonic solids are symmetrical isohedra (same faces), as players of role-playing games know. A fair die can't be made with an odd number of faces. Under a different definition of fair (not requiring congruent faces), there can be a clever trick to create a "fair" die which comes up with numbers 1–5 with equal probability. `https://en.wikipedia.org/wiki/Dice` □

19. In *graph theory*, a graph is composed of vertices (dots) and edges (lines). What is the minimum number of vertex colors required so that no two connected vertices in the graph shown share the same color?



- (A) 1
- $\boxed{\text{(B)}}$ 2
- (C) 3
- (D) 4
- (E) 5

SOLN This graph is called a bipartite graph; it can be characterized as a graph whose minimum vertex coloring is two because the graph can be decomposed into two sets of vertices X and Y where vertices in X are not adjacent (connected) to each other and vertices in Y are not adjacent to each other. □

20. The period T of a pendulum is the time it takes to make one complete oscillation. The period is related to the length L and the acceleration due to gravity $g = 9.8\,\mathrm{m/s^2}$ by

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

How long should the pendulum be in order to have a period of $T = \pi$ seconds?

- (A) 1 m
- (B) 1.5 m
- $\boxed{\text{(C)}}$ 2.45 m
- (D) 9.8 m
- (E) 19.6 m

SOLN Solving the period formula gives $L = \left(\frac{T}{2\pi}\right)^2 \cdot g = \left(\frac{\pi\,\mathrm{s}}{2\pi}\right)^2 \cdot 9.8\,\mathrm{m/s^2} = \frac{9.8\,\mathrm{m/s^2}}{4\,\mathrm{s^{-2}}}$ □