Bubble in the single best choice for each question you choose to answer.

1. If $a + b = 3$ and $a^2 + b^2 = 89$, then what is $a^3 + b^3$?

   (A) 307

   (B) 347

   (C) 387

   (D) 507

   (E) Not possible to determine

   > SOLN $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ so $a^3 + b^3 = 3(89 - ab)$. Note $9 = (a+b)^2 = (a^2 + 2ab + b^2)$, so $ab = -40$. □

2. Today €1.00 = \$1.25. If the value of the euro in dollars increases by 10% tomorrow, how many euros will 2.20 dollars be worth?

   (A) €1.50

   (B) €1.58

   (C) €1.60

   (D) €1.76

   (E) €1.94

   > SOLN $\dfrac{€1.00}{\$1.25 \times 1.10} \times \$2.20 = €\dfrac{1}{1.25}(2)$ □

3. The scale on an architectural floor plan is $1\,\text{in} : 12\,\text{ft}$. The length of a hallway in the floor plan is 1.75 in. What is the actual length of the hallway?

   (A) 15 ft

   (B) 18 ft

   (C) 21 ft

   (D) 24 ft

   (E) 27 ft

   > SOLN Use ratios. $\frac{1\,\text{in}}{12\,\text{ft}} = \frac{1.75\,\text{in}}{x} \implies x = (12\,\text{ft})(1.75) = 21\,\text{ft}$ □

4. A party-length sandwich that is 59 in long is cut into three pieces. The middle piece is 5 in longer than the shortest piece, and the shortest piece is 9 in shorter than the longest piece. How long is the longest piece?

   (A) 24 in

   (B) 20 in

   (C) 15 in

   (D) 59 in

   (E) 29 in
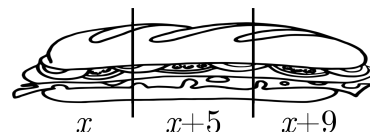
   $x \qquad x+5 \qquad x+9$

   > SOLN Call the shortest piece x.
   > $x + (x + 5) + (x + 9) = 59 \Rightarrow x = 15$ □

5. If $a \natural b = 2b - a$, then what is $(3 \natural 2) \natural 4$?

   (A) 3

   (B) 5

   (C) 7

   (D) 9

   (E) 11

   > SOLN $3 \natural 2 = 2(2) - 3 = 1$ and $1 \natural 4 = 2(4) - 1 = 7$ □

6. A circle and a square have the same perimeter. Then

   (A) their areas are equal.

   (B) the area of the circle is the greater.

   (C) the area of the square is the greater.

   (D) the area of the circle is π times the area of the square.

   (E) None of these

   > SOLN
   > $$A_\circ = \frac{P^2}{4\pi} \quad > \quad A_\square = \frac{P^2}{16}$$
   > For a given perimeter, a circle encloses the most area of any shape. □

7. Find the measure of an angle that is both the complement of $\angle A$ and the supplement of $\angle B$ if $m\angle A + m\angle B = 236°$.

(A) 17°

(B) 34°

(C) 45°

(D) 59°

(E) 67.5°

> SOLN Let $\angle C$ be the angle in question. Then $m\angle A + m\angle C = 90°$ and $m\angle B + m\angle C = 180°$. $m\angle A + m\angle C + m\angle B + m\angle C = 270°$. $2m\angle C = 270° - 236°$ □

8. Tau, who loves eating 2 pieces of pi, discovered that when the digits of a three-digit natural number are rearranged to form a second number, the difference between the two numbers is usually divisible by ____.

(A) 2

(B) 4

(C) 5

(D) 6

(E) 9

> SOLN Original number: $abc = 100a + 10b + c$
> If $bac = 100b + 10a + c$ $\Delta = 90a - 90b$
> If $bca = 100b + 10c + a$
> $\Delta = 99a - 90b - 9c$
> If $cab = 100c + 10a + b$
> $\Delta = 90a + 9b - 99c$
> If $cba = 100c + 10b + a$ $\Delta = 99a - 99c$
> If $acb = 100a + 10c + b$ $\Delta = 9b - 9c$ □

9. KSNO gives traffic reports every 10 minutes, 24 hours a day, but advertises "1000 reports each week." What is the difference between the exact number of reports and the advertised number?

(A) 8

(B) 12

(C) 16

(D) 20

(E) 24

> SOLN Six times per hour or 144 per day, times 7 is 1008 in a week. □

10. Convert the repeating decimal into a fraction. After reducing to lowest terms, find the difference between the denominator and the numerator. $0.60\overline{60}$

(A) 13

(B) 33

(C) 39

(D) 47

(E) 60

> SOLN
> $$100x = 60.\overline{60}$$
> $$x = 00.\overline{60}$$
> $$99x = 60$$
> $$\implies x = \frac{60}{99} = \frac{20}{33}$$ □

11. Laura loves leftovers. On Monday she ate $\frac{1}{4}$ of the leftover lasagna; on Tuesday, $\frac{1}{3}$ of what remained then; on Wednesday, $\frac{1}{2}$ of what remained then; and on Thursday she finishes it off. On what day did she eat more than on any other day?

(A) Monday

(B) Tuesday

(C) Wednesday

(D) Thursday

Mon	Tue
Thr	Wed

(E) She eats the same amount each day

> SOLN Call L the amount left over before Monday's dinner. She eats $\frac{L}{4}$ that day and $\frac{3L}{4}$ remains. On Tuesday she eats $\frac{1}{3}$ of that which is $\frac{L}{4}$ again, and $\frac{L}{2}$ remains. On Wednesday, she eats $\frac{1}{2}$ of that, which is $\frac{L}{4}$ again, and $\frac{L}{4}$ remains, which she eats on Thursday. □

12. A sphere is inscribed in a cube. What is the ratio of the volume of the sphere to the volume of the cube?

(A) $\frac{\pi}{6}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{8}$

(D) $\frac{6}{\pi}$

(E) $\frac{\pi}{2}$

$2r$
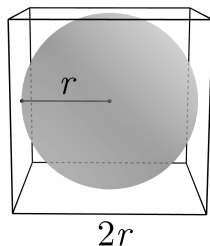
> SOLN Let r be the radius of the sphere. The sphere volume is $\frac{4}{3}\pi r^3$ and the cube volume will be $(2r)^3 = 8r^3$. The ratio of the sphere volume to the cube volume will be $\frac{\frac{4}{3}\pi r^3}{8r^3} = \frac{\pi}{6}$. □

13. Al and Bob are walking together at the same pace. Al stops to talk to Carl while Bob continues on at the same pace. After conversing with Carl, Al continues on at the same pace as before. He later passes Bob, who stopped to sit on a bench. How long has Bob been sitting on the bench when Al passes by?

(A) half as long as Al talked to Carl

(B) exactly as long as Al talked to Carl

(C) twice as long as Al talked to Carl

(D) four times as long as Al talked to Carl

(E) undeterminable

> SOLN Al completes two tasks (talking to Carl, then walking to the bench) in the same time that Bob completes two tasks (walking to the bench, then sitting on it). Since the walking pace is the same for both, the times spent walking to the bench are the same. Hence, Bob has been sitting on the bench for the same amount of time Al talked to Carl. □

14. The ages of 4 family members are represented by the positive integers a, b, c, and d, where $a < b < c < d$. Their mean age is 34, the median is 33, and the range of ages is 32. What is the value of a?

(A) 12

(B) 14

(C) 16

(D) 17

(E) 19

> SOLN From the mean: $a + b + c + d = 136$. From the median: $b + c = 2(33) = 66$. $\therefore a + d = 136 - 66 = 70$. The range gives $d - a = 32$. Subtracting the last two gives $2a = 38$. □

15. You have an account into which you place S dollars. At the end of each **even** year, you notice that your account has gained 50% in value compared to the previous year. And every **odd** year, your account loses 50% relative to the previous year. You first invest at the start of 2018. What is the value of your investment at the end of 2025?

   (A) $\frac{1}{2}S$

   (B) S

   (C) $\frac{27}{64}S$

   (D) $\frac{3}{2}S$

   (E) $\frac{81}{256}S$

   $\boxed{\text{SOLN}}$ Each even year the account gains 50%, so the account total is $1.5S$ at the end of 2018. At the end of 2019 the account value becomes $0.5 \times (1.5)S = 0.75S = \frac{3}{4}S$. In general, the value in the account is $\left(\frac{3}{4}\right)^n$ compared to the original value, where n is the number of *pairs* of years that have passed. The end of 2025 marks eight elapsed years, so $n = 4$ and the account value is $\left(\frac{3}{4}\right)^4$. This is not the kind of investment you want.   □

16. A train traveling at a constant speed takes 25 s from the time that it enters a tunnel 100 m long until the last car exits the tunnel on the other side. The last car is completely in the tunnel 5 s after the first car entered. How long is the train?

   (A)   15 m

   (B)   20 m

   (C)   25 m

   (D)   35 m

   (E)   50 m

   $\boxed{\text{SOLN}}$ $\frac{100+x}{25} = \frac{x}{5}$ is solved by $x = 25$.   □

17. How many whole numbers lie between $\sqrt{\pi}$ and π^2?

   (A)   4

   (B)   5

   (C)   6

   (D)   7

   (E)   8

   $\boxed{\text{SOLN}}$ $\sqrt{\pi} \approx 1.8$ and $\pi^2 \approx 9.87$.   □

18. Towns A, B, and C are at the corners of a triangle with equal sides. A car travels at constant speed from A to B at 30 mph, from B to C at 40 mph, and from C back to A at 60 mph. What is the average speed for the round trip?

   (A)   40 mph

   (B)   43 mph

   (C)   45 mph

   (D)   48 mph

   (E)   50 mph

   $\boxed{\text{SOLN}}$ The answer is the same for any size triangle, but let's assume a specific case for ease of illustration: $s = 120$ mi, so the total trip is 360 mi. The first leg takes $(120 \text{ mi})/(30 \text{ mi/h}) = 4$ h; likewise, the second leg takes 3 h, and the last leg takes 2 h, for a total of 9 h. The average speed is $360 \text{ mi}/9 \text{ h} = 40 \text{ mi/h}$.

   Marilyn vos Savant in <u>Parade</u>, Dec. 17, 2017.   □

19. Car A is $2\,\text{mi}$ ahead of car B, which is going in the same direction. $8\,\text{min}$ later car A is only $1\,\text{mi}$ ahead of car B. On average, how much faster is car B traveling?

(A) $5\,\text{mph}$

(B) $7.5\,\text{mph}$

(C) $10\,\text{mph}$

(D) $15\,\text{mph}$

(E) not enough info

SOLN Let d_i and v_i be the distance and speed car$_i$ travels in the $8\,\text{min}$. Then $d_A = 8v_A$ and $d_B = 8v_B = d_A + 1$. Substitute and subtract to get $8(v_B - v_A) = 1 \Rightarrow (v_B - v_A) = \frac{1}{8}\frac{\text{mi}}{\text{min}} = \frac{60}{8}\,\text{mph}$.
Shorter: Car B travels $1\,\text{mi}$ more in $8\,\text{min}$: $\frac{1}{8}\frac{\text{mi}}{\text{min}} = \frac{60}{8}\,\text{mph} = 7.5\,\text{mph}$ □

20. During shooting practice, a basketball player takes one step closer if she misses a shot, and one step farther away if she makes a shot. After a while, she notices she is two steps farther away than when she began. What is the most we can say about her shooting percentage P (i.e., shots made $\div$ shots taken)?

(A) $25\% < P \le 50\%$

(B) $P > 50\%$

(C) $P > 67\%$

(D) $P > 75\%$

(E) not enough info

SOLN If she's farther away, then $P > 50\%$. Without knowing how many shots she took, however, there's no way to know how much better than 50%. If she took just two shots $P = 100\%$; if she took thousands, P is slightly above 50%. □