Snow College Jr. Mathematics Contest

key

April 4, 2017 **Junior Division: Grades 7–9** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Boris is driving on a remote highway. He notices the odometer reading, 24942 km, is a palindromic number, meaning it is not changed when reversed. "Hmm," he thinks, "it should be a long time before I see that again." But it only takes one hour for the odometer to once again show a palindromic number! How fast is Boris driving?

   (A) 90 km/h
   (B) 100 km/h
   (C) 110 km/h
   (D) 120 km/h
   (E) 140 km/h

   SOLN The next palindromic number is 25052, so he has gone 25052 km − 24942 km in one hour. □

2. The minute hand of a courthouse clock measures 12 ft from center. How far, in feet, does the tip of the hand travel in 25 minutes?

   (A) 5π
   (B) 7π
   (C) 8π
   (D) 10π
   (E) 12π

   SOLN Each five-min increment has an angle of $\frac{2\pi}{12}$ rad $= \frac{\pi}{6}$ rad so in 25 min the hand has rotated $\frac{5\pi}{6}$ rad. The tip travels $12\,\text{ft} \cdot \frac{5\pi}{6}\,\text{rad} = 10\pi\,\text{ft}$.
   Soln 2: The path length of the tip over an hour (a complete revolution) is the circumference $2\pi r = 2\pi(12\,\text{ft})$. We want 5/12 of that. □

3. Dale lives 0.6 mi from school and it takes him 10 min to walk to school. Marc can walk to school in 5 min and he lives 0.3 mi from school. Jeremy lives 1.5 mi from school, and it takes him 25 min to walk to school while it takes Brittany 30 min to walk 1.8 mi to school. All of the following conclusions can be drawn from the given data EXCEPT:

   (A) They all walk at the same speed.
   (B) They all live < 2 mi from school.
   (C) Dale lives closer to school than Brittany, but farther than Marc.
   (D) Brittany and Jeremy walk slower than Dale and Marc.
   (E) It takes Brittany 3 times as long as Dale to walk to school.

   SOLN Check rate = dist/time for each. □

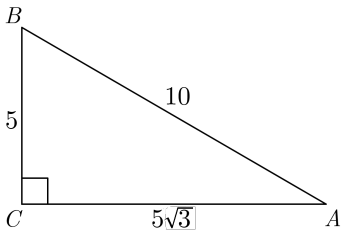
4. If $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ is continued, what is the denominator of the 10th term?

   (A) 64
   (B) 212
   (C) 256
   (D) 512
   (E) 1024

   SOLN The denominator of the nth term is 2^n, and $2^{10} = 1024$. □

5. What is the measure of angle A?

(A) $30°$
(B) $45°$
(C) $60°$
(D) $75°$
(E) $90°$



SOLN Recall that if short leg is half the hypotenuse the smallest angle is $30°$. □

6. A base 26 number system is used with the letters of the alphabet as the digits: A = 0, B = 1, C = 2, ... Y = 24, Z = 25. In this system, what is ONE + ONE?

(A) DBIA
(B) BAID
(C) BDAI
(D) DAIB
(E) BDIA

SOLN The least significant digit is sufficient: E = 4, so E + E = 8, and I = 8. □

7. What is the smallest positive integer n such that $1 + 2 + 3 + \ldots + n > 5000$?

(A) 90
(B) 99
(C) 100
(D) 101
(E) 110

SOLN The sum of the first n positive integers is $\frac{n(n+1)}{2}$. The solution is the first positive n for which $n(n+1) > 10000$. When is the product of consecutive positive integers first larger than $10000 = 100^2$? Clearly, $99 \cdot 100 < 100 \cdot 100 < 100 \cdot 101$; therefore $n = 100$. □

8. The city council has changed the numbering scheme for the 200 houses on Elm Street. They will be renumbered with the natural numbers from 1 through 200. A city worker is given a box of 1000 metal numbers, 100 of each digit, and told to distribute new house numbers in order starting with 1 Elm Street. What is the first address for which he will not have the correct digits?

(A) 137 Elm Street
(B) 163 Elm Street
(C) 172 Elm Street
(D) 191 Elm Street
(E) 199 Elm Street

SOLN The first metal 1 will be used for address 1, the next eleven 1s for addresses 10–19, the next eight for 21–91, the next eleven for 100–109, the next twenty-one for 110–119, and another eleven each for 120–129, 130–139, 140–149, 150–159, and the last four for 160–162. □

9. One can of frozen juice concentrate, when mixed with $4\frac{1}{3}$ cans of water, makes 2 quarts (64 oz) of juice. Assuming no volume is gained or lost by mixing, how many oz does a can hold?

(A) 8
(B) 10
(C) 12
(D) 15
(E) 18

SOLN $5\frac{1}{3}$ cans of concentrate and water makes 64 oz; solve the ratio for x.

$$\frac{\frac{16}{3}\,\text{cans}}{64\,\text{oz}} = \frac{1\,\text{can}}{x\,\text{oz}}$$

□

10. Suzie wants to draw eight 1-inch wide stripes on a 24-inch-wide poster. Both sides of the poster will have a stripe right on the edge. She wants the gaps between the eight stripes to be the same everywhere. What is the width of each gap between the stripes?

(A) 3 in
(B) $\frac{16}{7}$ in
(C) 2 in
(D) $\frac{7}{4}$ in
(E) $\frac{8}{7}$ in

SOLN There will be 7 gaps. Of the 24 inches on the poster, 8 will be covered with stripes leaving 16 inches for 7 gaps. □

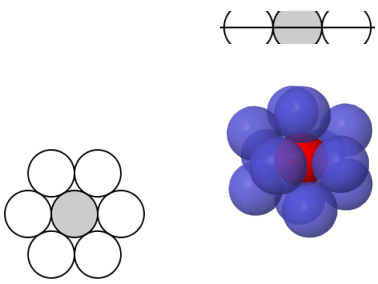
11. Two straight lines have the same y-intercept and reciprocal slopes. If the first line has slope m and x-intercept a, what is the x-intercept of the other line?

(A) $\frac{m^2}{a}$
(B) $\frac{m}{a}$
(C) $\frac{1}{a}$
(D) am
(E) am^2

SOLN The equation of the first line is $y = m(x - a)$ so the y-intercept is $-ma$. Therefore the equation of the second line is $y = \frac{1}{m}x - ma$. Setting $y = 0$, $x = am^2$. From AMATYC Faculty Math League. □

12. In geometry, a *kissing number* is the number of non-overlapping unit spheres that can be arranged such that they each touch a given unit sphere. In 1-D, the kissing number is 2. In 3-D the kissing number is 12. What is the kissing number in 2-D?

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8



SOLN https://en.wikipedia.org/wiki/Kissing_number_problem
There is almost enough room left over in 3-D to fit in a 13th, but not quite. □

13. Simplify $\sqrt{\sqrt[0.06]{2^{0.12}}}$

(A) 1
(B) $\sqrt{2}$
(C) 2
(D) $2^{0.0384}$
(E) 4

SOLN Writing with rational exponents gives $((2^{0.12})^{1/0.06})^{1/2} = 2^{(0.12)(1/0.06)(1/2)} = 2^1 = 2$. □

14. In a drawer are 6 black, 2 gray, and 2 tan socks. What is the probability of blindly pulling out (without replacement) four black socks in a row?

(A) $\frac{1}{14}$
(B) $\frac{1}{7}$
(C) $\frac{1}{4}$
(D) $\frac{2}{5}$
(E) $\frac{3}{5}$

SOLN To find the probability, multiply $(\frac{6}{10})(\frac{5}{9})(\frac{4}{8})(\frac{3}{7}) = (\frac{3}{5})(\frac{5}{9})(\frac{1}{2})(\frac{3}{7})$ □

15. Given sets $A = \{\text{evens}\}$, $B = \{\text{non-primes}\}$, and $C = \{\text{primes} < 19\}$, and universal set $U = \{0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23\}$, find the complement of $A \cup B \cup C$.

(A) $\{0, 23\}$

(B) $\{0, 10, 20\}$

(C) $\{23\}$

(D) $\{1, 3, 11, 13, 21, 23\}$

(E) $\emptyset$

SOLN $A \cup B = \{0, 2, 10, 12, 20, 21, 22\}$. Then the union of this set with C is $\{0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22\}$ the complement in U of which is $\{23\}$. □

16. In a music class, 15 students play violin, 14 play piano, and 16 play horns. Of these, 6 play piano and violin, 9 play piano and horns, and 5 play horns and violin. Four students play all three and 2 students play none. How many students are in the class?

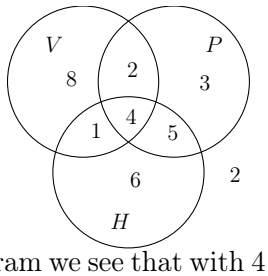
(A) 21

(B) 23

(C) 31

(D) 45

(E) 47



SOLN On a Venn diagram we see that with 4 students who play all three instruments there are 2 who play only violin and piano, 5 who play only piano and horns, and 1 who plays only violin and horns. From there, we see that 8 play violin only, 3 play piano only, and 6 play horns only. With the 2 who do not play any instruments, we have a total of 31. □

17. Square $PQRS$ has sides of length 10. Points T, U, V, and W are chosen on sides PQ, QR, RS, and SP respectively so that $PT = QU = RV = SW = 2$. Find the area of quadrilateral $TUVW$.

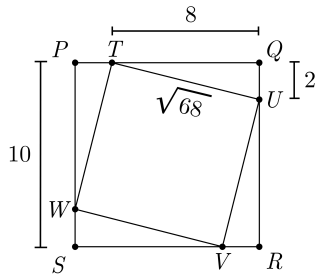
(A) 48

(B) 52

(C) 56

(D) 64

(E) 68



SOLN You have a square within a square whose sides form the equal hypotenuses of 4 right triangles with legs 2 and 8. Each side of the inner square is $\sqrt{68}$. □

18. Pi High School sent a team of 5 students to the Snow College Math Contest in 2015. In 2016 PHS sent the same team except the oldest member (graduated) was replaced with a younger student. If the average team member age was the same for both years, how many years younger was the new member than the old member who was replaced?

(A) 1

(B) 2

(C) 3

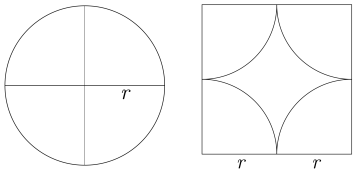
(D) 4

(E) 5

SOLN $$\frac{a_1 + a_2 + a_3 + a_4 + a_{\text{old}}}{5} = \frac{(a_1 + a_2 + a_3 + a_4 + 4) + a_{\text{new}}}{5}$$

□

19. What is the ratio of the area of a circle to the area of the figure created by flipping each quarter circle around its chord?



(A) $\frac{3\pi}{4}$

(B) $\pi + 1$

(C) π

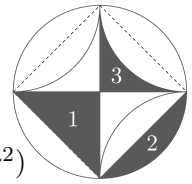
(D) $\pi - 1$

(E) $\frac{\pi}{4-\pi}$

SOLN $A_\diamondsuit = A_\square - A_\bigcirc = (2r)^2 - \pi r^2$

$$\frac{A_\bigcirc}{A_\diamondsuit} = \frac{\pi r^2}{4r^2 - \pi r^2} = \frac{\pi}{4-\pi}$$

Method 2:



$A_1 = \frac{1}{2}r^2$

$A_2 = \frac{1}{4}\pi r^2 - \frac{1}{2}r^2$

$A_3 = \frac{1}{2}r^2 - (\frac{1}{4}\pi r^2 - \frac{1}{2}r^2)$

$= r^2 - \frac{1}{4}\pi r^2 = r^2(1 - \frac{\pi}{4})$

$$\frac{A_\bigcirc}{4A_3} = \frac{\pi r^2}{4r^2(1-\frac{\pi}{4})} = \frac{\pi}{4-\pi}$$

□

20. Definition of the *triangle of power* notation:

$$\begin{matrix} & y & \\ x & \triangle & z \end{matrix} \Leftrightarrow x^y = z \Leftrightarrow \sqrt[y]{z} = x \Leftrightarrow \log_x z = y$$

Any of x, y, z is equivalent to the triangle of power with that number missing, e.g.,

$$\begin{matrix} & y & \\ & \triangle & \\ & z & \end{matrix} = x$$

Find the value of the following. $\begin{matrix} 2 & \triangle & 8 \end{matrix}$

(A) 2

(B) $\sqrt{8}$

(C) 3

(D) 4

(E) 16

SOLN $2^3 = 8,\ \sqrt[3]{8} = 2,\ \log_2 8 = 3$ □