Snow College Jr. Mathematics Contest

key

April 5, 2016 **Junior Division: Grades 7–9** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Home plate on a baseball field has a shape that is a square with two congruent isosceles right triangles removed from adjacent corners. What is the measure of $\angle ABC$?

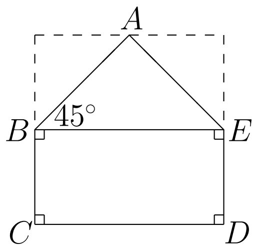
   (A) 105°

   (B) 110°

   (C) 115°

   (D) 120°

   $\boxed{\text{(E)}}$ 135°



   SOLN $m\angle EBC = 90^\circ$, $m\angle ABE = 45^\circ$. □

2. Using a scale, you find that four t**w**eezers and a **b**ar of soap weigh the same as three **c**ombs; a bar of soap weighs the same as two **t**oothbrushes and a comb; and six tweezers weigh the same as one toothbrush and one comb. How many combs are needed to equal the weight of one bar of soap?

   $\boxed{\text{(A)}}$ 2

   (B) 4

   (C) 6

   (D) 8

   (E) 10

   SOLN Let w be tweezers, t toothbrushes. (1) $4w + b = 3c$, (2) $b = 2t + c$, and (3) $6w = t + c$. Eliminating w from the system made up of equations (1) and (3) gives $0 = 7c - 3b - 2t$. Eliminating t from this new equation and equation (2) gives $b = 2c$. □

3. If Gretchen's house is 15 miles due west of Lino's house and 20 miles north of Isaac's house, how far is Lino's house from Isaac's house as the crow flies (direct route)?

   (A) 12 miles

   $\boxed{\text{(B)}}$ 25 miles

   (C) 27 miles

   (D) 30 miles

   (E) 32 miles

   SOLN Set up a right triangle with legs 15 and 20. Recognize 3-4-5 triangle increased by a factor of 5, or use Pythagorean theorem. □

4. A fountain spouts water from two eyes, a mouth, and a foot. The right eye would fill a specific jar in two days, the left eye in three days, the mouth in six hours, and the foot in four days. To the nearest hour, how long will it take all four of these spouts together to fill the jar?

   (A) 2

   (B) 3

   (C) 4

   $\boxed{\text{(D)}}$ 5

   (E) 6

   SOLN $\left(\frac{1}{2} + \frac{1}{3} + 4 + \frac{1}{4}\right) t = 1$, where t is the time required to fill the jar using all spouts. Solving gives $t = \frac{12}{61}\,\mathrm{d} \approx \frac{12}{60}\,\mathrm{d} = \frac{1}{5}\,\mathrm{d} \approx \frac{5}{25}\,\mathrm{d} \approx \frac{5}{24}\,\mathrm{d} \approx 5\,\mathrm{h}$. Alt soln: LCM $= 12\,\mathrm{d}$. In $12\,\mathrm{d}$ the right eye fills 6 jars, left eye 4 jars, mouth 48 jars, and foot 3 jars. This is 61 jars in 12 days or $12/61$ days per jar. (Adapted from Greek Anthology by Metrodorus circa 500 A.D.) □

5. What is $2^{50} + 2^{50} + 3^{50} + 3^{50} + 3^{50}$?

   (A) 5^{50}

   $\boxed{\text{(B)}}$ $2^{51} + 3^{51}$

   (C) 5^{51}

   (D) $2^{100} + 3^{150}$

   (E) 5^{100}

   $\boxed{\mathcal{SOLN}}$ $2 \cdot 2^{50} + 3 \cdot 3^{50} = 2^{51} + 3^{51}$ □

6. Marc owns 7 acres of land. He grows tomatoes on $\frac{5}{6}$ of his land. On how many acres of land does Marc grow tomatoes?

   (A) 5 acres

   (B) $3\frac{1}{2}$ acres

   (C) $4\frac{2}{3}$ acres

   (D) 6 acres

   $\boxed{\text{(E)}}$ $5\frac{5}{6}$ acres

   $\boxed{\mathcal{SOLN}}$ $\frac{5}{6} \times 7 = \frac{35}{6} = 5\frac{5}{6}$ □

7. While cleaning out the garage, Larry found four old single-digit house numbers: one 3, one 4, and two 5s. How many different two-digit house numbers can he create?

   (A) 4

   (B) 5

   (C) 6

   $\boxed{\text{(D)}}$ 7

   (E) 8

   $\boxed{\mathcal{SOLN}}$ Brute force gives 34, 35, 43, 45, 53, 54, 55. Better: see each combination has distinct digits except 55. For those with distinct digits, there are 3 ways to choose the first digit and 2 ways to choose the second for $3! = 6$ ways. Add the 55 case to get a total of 7. □

8. What is the measure of an interior angle of a regular heptagon?

   $\boxed{\text{(A)}}$ $\frac{900}{7}^{\circ}$

   (B) 140°

   (C) $\frac{360}{7}^{\circ}$

   (D) 40°

   (E) 120°

   $\boxed{\mathcal{SOLN}}$ The vertex angle measure of a regular n-gon is $(n-2)\frac{180^{\circ}}{n}$. □

9. Consider the equation $x^2 + bx + 2 = 0$. A single, fair, 6-sided die is rolled to determine the value of the middle coefficient b which becomes the number of pips on the upper face of the die. What is the probability that the equation will have real unequal roots?

   (A) $\frac{1}{6}$

   (B) $\frac{1}{3}$

   (C) $\frac{1}{2}$

   $\boxed{\text{(D)}}$ $\frac{2}{3}$

   (E) $\frac{5}{6}$

   $\boxed{\mathcal{SOLN}}$ The discriminant of the quadratic equation is $\sqrt{b^2 - 8}$ and is a real non-zero number for $b = 3, 4, 5, 6$. □

10. In 6 years Bill's age will be a perfect square. Six years ago Bill's age was the square root of that perfect square. How old is Bill?

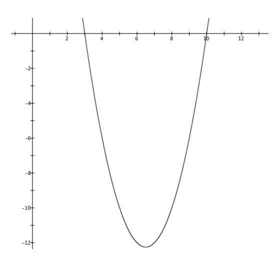
    (A) 6

    (B) 8

    $\boxed{\text{(C)}}$ 10

    (D) 12

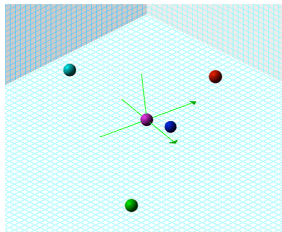
    (E) 14

    $\boxed{\mathcal{SOLN}}$ Solve $a+6 = (a-6)^2$ where $a - 6 \geqslant 0$.
    $a^2 - 13a + 30 = 0$ □

11. How many ordered triples (x, y, z) of real numbers satisfy the conditions?

$$xy = z, \quad xz = y, \quad yz = x$$



(A) 1

(B) 2

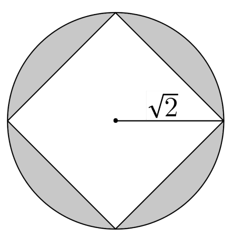
(C) 3

(D) 4

$\boxed{\text{(E)}}$ 5

SOLN If one of x, y, z is 0 then all three are, yielding triple $(0, 0, 0)$. If none are 0 then each of x, y, z must be ± 1. $(1, 1, 1)$ works. If any is negative then exactly two of them are: $(1, -1, -1)$, $(-1, 1, -1)$, $(-1, -1, 1)$. $\square$

12. A square is inscribed in a circle with radius $\sqrt{2}$. Find the area of the region of the circle not included in the square.



(A) $2\sqrt{2}\pi - 4$

(B) $\pi^2 - 2$

$\boxed{\text{(C)}}$ $2\pi - 4$

(D) $2\pi - 2$

(E) $4 - 2\pi$

SOLN $A_\circ = \pi r^2 = 2\pi$. A_{rhombus} is half product of diagonals: $\frac{(2\sqrt{2})(2\sqrt{2})}{2} = 4$ $\square$

13. $\frac{68}{77} = \frac{a}{7} + \frac{b}{11}$, $\quad 0 < a < 7, \quad 0 < b < 11$ What is $a + b$?

(A) 6

$\boxed{\text{(B)}}$ 8

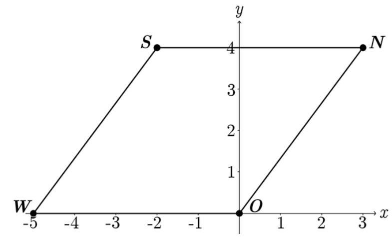
(C) 11

(D) 17

(E) $8\sqrt{5}$

SOLN $11a + 7b = 68 \qquad \frac{68}{77} = \frac{3}{7} + \frac{5}{11}$ $\square$

14. Find the area, in square units, of the quadrilateral $SNOW$ where $S = (-2, 4)$, $N = (3, 4)$, $O = (0, 0)$, and $W = (-5, 0)$.



$\boxed{\text{(A)}}$ 20

(B) 18

(C) 25

(D) $8\sqrt{5}$

(E) $16\sqrt{2}$

SOLN $SNOW$ is a parallelogram: $A = bh$. With $SN = 5$ and a height of 4, $A = 20$. Alternatively, $SNOW$ is a rhombus, and the area of a rhombus is half the product of the diagonals. $NW = \sqrt{80}$ and $SO = \sqrt{20}$, so $A = \frac{1}{2}\sqrt{20 \cdot 80} = \frac{1}{2}\sqrt{1600}$. $\square$

15. The stem and leaf plot shows the scores of the last test in Mr. Newton's math class. Which statement about the scores is true?

stem	leaf
9	2 5 8
8	0 0 1 3 4 7 7 9
7	5 8 8
6	3 6

$\boxed{\text{(A)}}$ More than 50% scored above an 80.

(B) The highest score was a 92.

(C) The median score was an 80.

(D) The range of scores was 50.

(E) The mode score was 81.

SOLN The stem column is the 10s place and the leaf column is the 1s place, so the first row shows scores of 98, 95, and 92. There were 9 scores above 80. $\square$

16. Two objects are topologically equivalent if we can stretch, shrink, bend, or twist one, without cutting or gluing, and deform it into the other. Which is equivalent to **F** ?

(A) **G**

(B) **L**

(C) **W**

(D) **X**

$\boxed{\text{(E)}}$ **Y**

SOLN The branches of **Y** can be transformed into the branches of **F**. Each has one point from which you can go in three directions. □

17. At the G8 summit, the top leaders of 8 nations meet to discuss crucial political topics. If each of the 8 leaders formally greets the other 7 leaders with a handshake, how many handshakes take place?

(A) 7

(B) 8

$\boxed{\text{(C)}}$ $28 = {}_8C_2$

(D) $7! = 5040$

(E) $8! = 40320$

SOLN The first leader shakes hands with 7 others; the second with 6 others, and so on. $7+6+5+4+3+2+1$ □

18. Blims vary inversely as yops squared. If 100 blims go with 2 yops, how many blims go with 10 yops?

(A) 2

$\boxed{\text{(B)}}$ 4

(C) 200

(D) 400

(E) 2500

SOLN $B = \frac{k}{Y^2}$ $\quad 100 = \frac{k}{2^2} \implies k = 400$
Soln 2: $\frac{B_1}{B_2} = \frac{Y_2^2}{Y_1^2}$ $\quad \frac{100}{B_2} = \frac{10^2}{2^2}$ □

19. Evaluate. $\quad (1+\mathrm{i})^2$

(A) $-2-\mathrm{i}$

(B) $-1+\mathrm{i}$

$\boxed{\text{(C)}}$ $2\mathrm{i}$

(D) $1-2\mathrm{i}$

(E) $2+2\mathrm{i}$

SOLN $1\cdot 1 + 1\cdot \mathrm{i} + \mathrm{i}\cdot 1 + \mathrm{i}\cdot\mathrm{i} = 1 + 2\mathrm{i} - 1$
Soln 2: If $x, y \in \mathbb{C}$ and $z = xy$, then $\angle z = \angle x + \angle y$ and $|z| = |x|\,|y|$. $\angle(1+\mathrm{i}) = 45^\circ$ and $|1+\mathrm{i}| = \sqrt{2}$ □

20. The lines $2x - y = a$ and $y - a = b$ intersect at the point (p, q). Find q.

$\boxed{\text{(A)}}$ $a+b$

(B) $2a+b$

(C) $a-b$

(D) $a+2b$

(E) $2a-b$

SOLN The second line is horizontal at a height of $y = a+b$, so the intersection must have that y-value. □