Snow College Jr. Mathematics Contest

key

April 1, 2014 **Junior Division: Grades 7–9** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Maria owns a bicycle shop and ordered 1200 parts; 60 were found to be defective. What percent of the parts were defective?

   (A) 2%
   (B) 3%
   (C) 4%
   (D) 5%
   (E) 6%

   soln $\frac{60}{1200} = \frac{6}{120} = \frac{1}{20} = 0.05$ □

2. Which of the following products is/are palindromic (read the same backwards and forwards)?

   (i) 11111×11111
   (ii) 22222×22222
   (iii) 33333×33333
   (iv) 44444×44444

   (A) only (i)
   (B) only (ii) and (iii)
   (C) only (iii) and (iv)
   (D) only (i), (ii), and (iv)
   (E) none of them

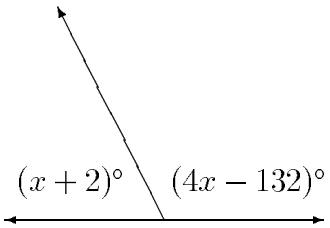
   soln Look at patterns:

   $$\begin{aligned} 1^2 &= 1 \\ 11^2 &= 121 \\ 111^2 &= 12321 \\ 1111^2 &= 1234321 \end{aligned}$$

   Note: (ii) is 4× (i) so it will cause non-palindromeness (because of carries/trades) in the thousands place value. Similary for (iii) and (iv). □

3. Find the measure of each marked angle.

   (A) 66° and 114°
   (B) 62° and 118°
   (C) 64° and 116°
   (D) 64° and 26°
   (E) 45° and 135°

   soln Supplementary angles.
   $(x+2) + (4x-132) = 180$
   $5x - 130 = 180$
   $5x = 310 \implies x = 62$ □

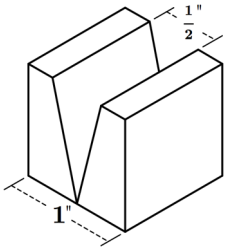
4. What is the average of the first 99 positive integers?

   (A) 49.00
   (B) 49.50
   (C) 49.75
   (D) 50.00
   (E) 50.25

   soln The sum is $\frac{(99)(1+99)}{2} = (99)(50)$, so the average is $\frac{(99)(50)}{99} = 50$. □

5. A wedge is removed from the center of a cube as shown. How much of the original volume of the cube remains?

   (A) $\frac{3}{5}$
   (B) $\frac{1}{2}$
   (C) $\frac{3}{4}$
   (D) $\frac{\sqrt{2}}{2}$
   (E) $\frac{4}{5}$

   soln $V_{\text{cube}} - V_{\text{wedge}} = 1 - \frac{1}{2}(\frac{1}{2} \cdot 1) = \frac{3}{4}$.
   Or turn over the piece on the right and join it to the piece on the left to produce a rectangular solid that is 3/4 as wide as the cube was. □

6. If you take two ordinary decks of cards (minus the jokers, for a total of 104 cards), thoroughly shuffle them, and then divide them into two equal piles, what are the chances that the number of red cards in pile 1 equals the number of black cards in pile 2?

[(A)] 100%
(B) 75%
(C) 50%
(D) 25%
(E) 0%

[SOLN] Call the number of red cards in pile 1 x. Then the number of black cards in pile 1 is $52 - x$. Then the number of black cards in pile 2 is $52 - (52 - x) = x$. From Car Talk Puzzler 29 Dec 2012. □

7. Inside one of three boxes is a unique gem. To help you find the gem there is an inscription on each box. The gold box says, "The gem is in this box." The silver box says, "The gem isn't in this box." The lead box says, "The gem isn't in the gold box." One, and only one, of these inscriptions is true. Which box contains the gem?

[(A)] silver
(B) gold
(C) lead
(D) none of the boxes
(E) Not enough information

[SOLN] It can't be the gold box or the gold and silver inscriptions would be true. It can't be the lead box or the silver and lead inscriptions would be true. From Car Talk Puzzler 5 Jan 2013. □

8. The two legs of a right triangle measure 9 in and 12 in. What is the perimeter?

(A) 15 in
(B) 21 in
(C) 25 in
(D) 30 in
[(E)] 36 in

[SOLN] The remaining side is 15 in. So the perimeter is 9 in + 12 in + 15 in. □

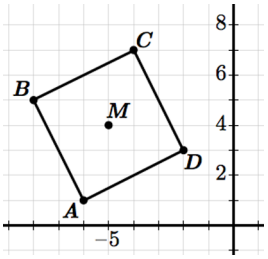
9. Which is equal to 153×10^{-4}?

(A) 0.000153
[(B)] 0.0153
(C) 0.153
(D) 1.53
(E) 1 530 000

[SOLN] Move the decimal four places to the left. □

10. In the square $ABCD$, point A is located at $(-6, 1)$ and point C is located at $(-4, 7)$. If the square is rotated around point M so that the new position of point A is $(-8, 3)$, what is the new position of point C?

(A) $(-4, 7)$
[(B)] $(-2, 5)$
(C) $(-2, 9)$
(D) $(-5, 5)$
(E) $(-6, 9)$



[SOLN] Point A moves 2 left and 2 up, so point C moves 2 right and 2 down. □

11. Amber drew a scale drawing of a game room. In real life the pool table is 4 feet wide. It is 8 inches in the drawing. What scale did Amber use for the drawing?
1 inch = _____ feet.

(A) $\frac{1}{4}$
(B) $\frac{1}{3}$
(C) $\frac{1}{2}$
(D) 1
(E) 2

SOLN Divide 8 in = 4 ft by 8. □

12. You roll a die and flip a coin. How many outcomes are possible?

(A) 6
(B) 8
(C) 10
(D) 12
(E) 14

SOLN With heads on the coin you get 6 choices for the die; likewise for tails. □

13. For the given data, what are the lower quartile, the median, and the upper quartile?

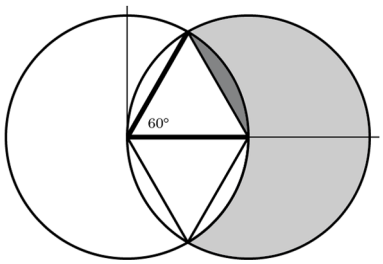
12 20 22 28 29 58 63

	Lower Quartile	Median	Upper Quartile
(A)	21	28	50
(B)	12	29	63
(C)	20	28	45.5
(D)	16	29	50
(E)	20	28	58

SOLN The median of an ordered set of numbers is the one in the middle. The middle one here is 28 since there are an odd number of numbers. The lower and upper quartiles are respectively the middle of each of the halves. □

14. What is the area shared by two intersecting circles of radius 1 passing through each other's center?

(A) $\frac{\sqrt{3}}{2}$
(B) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$
(C) $\frac{\sqrt{3}}{2} + \frac{\pi}{18}$
(D) $\frac{\pi}{4}$
(E) $\frac{\pi}{3}$



SOLN Geometry: Consider one wedge from the center of the circle on the left, the inscribed equilateral triangle, and the left-over lunate sliver. $A_{\text{wedge}} = \frac{\pi}{6}$ and $A_{\text{triangle}} = \frac{\sqrt{3}}{4}$, so $A_{\text{sliver}} = \frac{\pi}{6} - \frac{\sqrt{3}}{4}$. The total area is two equilateral triangles plus four slivers: $A_{\text{tot}} = 2 \cdot \frac{\sqrt{3}}{4} + 4 \cdot \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$. □

15. In a recent election for student body president, Linus received 3/5 of the total votes, Charlie received 2/7 of the total votes, and Lucy received the remaining 24 votes. How many votes did Charlie receive?

(A) 24
(B) 60
(C) 75
(D) 100
(E) 126

SOLN Linus and Charlie together received $\frac{31}{35}$ of the total votes. This implies that Lucy's 24 votes were $\frac{4}{35}$ of the total. $24 = \frac{4}{35}x \implies x = 210$ total votes. 2/7 of 210 is 60. □

16. A triangle has sides of lengths 8.1 and 1.4. What is the length of the third side, if it is an even integer?

(A) 2

(B) 4

(C) 6

(D) 8

(E) 10

SOLN $8.1 - 1.4 < c < 8.1 + 1.4 \implies 6.7 < c < 9.5$ □

17. If the radius of a circle is increased by 1, by how much is the circumference increased?

(A) 1

(B) 2

(C) 3

(D) π

(E) 2π

SOLN $C_1 = 2\pi R \quad C_2 = 2\pi(R+1)$
$C_2 - C_1 = 2\pi$ □

18. What is the product of the greatest common divisor of 9633 and 4693 and the least common multiple of the same numbers?

(A) 183 027

(B) 2 379 351

(C) 3 477 513

(D) 45 207 669

(E) 3 722 098 081

SOLN

$$\text{lcm}(m,n) = \frac{m \cdot n}{\gcd(m,n)} \implies$$

$$\text{lcm}(m,n) \cdot \gcd(m,n) = m \cdot n$$

Since m and n both end in 3 their product must end in 9, so the full multiplication is not necessary since all other choices are eliminated. □

19. Ms. Pham writes 2 tests, each with 25 problems. If the tests have 12 problems in common, how many problems does she write?

(A) 24

(B) 26

(C) 37

(D) 38

(E) 49

SOLN $13 + 13 + 12 = 38 = 2(25) - 12$ □

20. Four hat-wearing friends sat at a table discussing practical jokes for April Fool's Day. Over speaker-phone hatless Tim learned that the person wearing the cowboy hat sat on Frosty's left, and Rudolph sat opposite the person wearing the top hat. The Grinch was not wearing the beret and would not be caught dead in a cowboy hat. The person wearing the cowboy hat sat opposite from Charlie. Who was wearing the sombrero?

(A) Charlie

(B) Frosty

(C) Grinch

(D) Rudolph

(E) Tim

SOLN Since the one in the cowboy hat sat to the left of F and opposite C and was not G, then G must have sat opposite from F, and R wore the cowboy hat while sitting across from C. Thus, C must have worn the top hat, F wore the beret, and G must have been wearing the sombrero. □