Snow College Jr. Mathematics Contest

key

April 6, 2010 **Junior division: grades 7–9** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Allan is now twice as old as Gloria was three years ago. In three years, Gloria will be as old as Allan is now. How old is Gloria?

   (A) 3
   (B) 6
   [(C)] 9
   (D) 12
   (E) 15

   SOLN $G+3 = 2(G-3)$ □

2. In a certain family each daughter has the same number of brothers and sisters. Each son has twice as many sisters as brothers. How many children are in the family?

   (A) three
   (B) four
   (C) five
   (D) six
   [(E)] more than six

   SOLN Let g be the number of girls and b be the number of boys. The first sentence says that $g = b+1$. The second sentence says $g = 2(b-1)$. Setting the two equations for g equal to each other gives $b+1 = 2(b-1) \Rightarrow b = 3$.□

3. What is the sum of the exponents in the prime factorization of 2010?

   (A) 2
   (B) 3
   [(C)] 4
   (D) 7
   (E) 2010

   SOLN $2010 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 67^1$. □

4. The odds in favor of an event are 3 to 2. What is the probability of the event?

   (A) 0.2
   (B) 0.4
   [(C)] 0.6
   (D) 0.8
   (E) 1.5

   SOLN $\frac{3}{3+2} = 0.6$ □

5. An airport waiting room contains 80 people. If 15% of those people are not U.S. citizens, how many U.S. citizens are in the room? Assume exactly one nationality per person.

   (A) 12
   (B) 14
   (C) 56
   [(D)] 68
   (E) 70

   SOLN $(1-15\%)(80) = 0.85(80) = 68$ □

6. Fill in the blank.

I think Fibonacci is fun,
You start with a one and a one,
Then two, three, five, _____,
But don't stop there, mate,
The fun has just barely begun.

(A) straight

(B) seven

(C) eleven

(D) eight

(E) six

SOLN $F_{n+2} = F_n + F_{n+1}$. Check the number of syllables in each line. Thanks to Arthur Benjamin for this limerick. □

7. A *tesselation* or tiling of the plane is a collection of plane figures that fills the plane with neither overlaps nor gaps. Which regular polygon can**not** tile the plane?

(A) Equilateral triangle

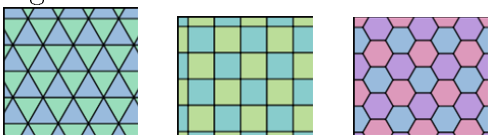
(B) Square

(C) Regular pentagon

(D) Regular hexagon

(E) All of the above can tile the plane.

SOLN Here are the only three possible regular tesselations in 2D:

 □

8. If the natural numbers are arranged in the following pattern what is the 7th number (from the left) in the 10th row?

$$\begin{array}{ccccccccccc}
1 & & 2 & & 5 & & 10 & & 17 & & \vdots \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
4 & \leftarrow & 3 & & 6 & & 11 & & 18 & & \\
 & & & & \downarrow & & \downarrow & & \downarrow & & \\
9 & \leftarrow & 8 & \leftarrow & 7 & & 12 & & 19 & & \\
 & & & & & & \downarrow & & \downarrow & & \\
16 & \leftarrow & 15 & \leftarrow & 14 & \leftarrow & 13 & & 20 & & \\
 & & & & & & & & \downarrow & & \\
25 & \leftarrow & 24 & \leftarrow & 23 & \leftarrow & 22 & \leftarrow & 21 & &
\end{array}$$

(A) 88

(B) 94

(C) 96

(D) 97

(E) 107

SOLN Note patterns like the perfect squares down the left. The number in the kth position of the nth row is

$$\begin{array}{ll} n^2 - k + 1 & \text{if } k \le n \\ (k-1)^2 + n & \text{if } k \ge n \end{array}$$

Extra: show that these two expressions are equal when $n = k$. □

9. In the table the sum of each row, column, and diagonal is the same.
What is the value of $A + B + C + D$?

(A) 80

(B) 64

(C) 96

(D) 60

(E) 72

A	4	B
10	16	22
C	28	D

SOLN The middle row sums to $10+16+22 = 48$. Therefore row 1 and row 3 have to each add to 48. This yields $(A+4+B)+(C+28+D) = 48+48$. Solving yields $A+B+C+D = 64$. □

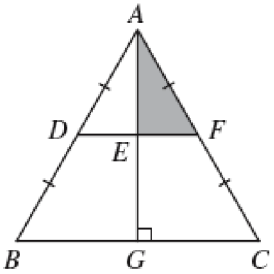
10. During the first three basketball games, Jordan scored an average of 18 points. After the fourth game, Jordan's scoring average dropped to 17 points. How many points did Jordan score in the fourth game?

(A) 14
(B) 15
(C) 16
(D) 17
(E) 18

SOLN With an average of 18 points per game for the first three games, Jordan scored a total of $3 \times 18 = 54$ points. Let x represent the number of points scored in game 4. $(54 + x)/4 = 17$. Solving for x yields $x = 14$. □

11. $\triangle ABC$ is isosceles with $AB = AC$, and $AG \perp BC$. Point D is the midpoint of AB, point F is the midpoint of AC, and E is the point of intersection of DF and AG. What fraction of the area of $\triangle ABC$ does the shaded area represent?

(A) 1/12
(B) 1/6
(C) 1/4
(D) 1/10
(E) 1/8



SOLN Let $x = EF$ and $y = AE$. Then the area of $\triangle AEF = \frac{1}{2}xy$.
By similar triangles, $GC = BG = 2x$, so $BC = 4x$. Similarly (good pun) $AG = 2y$. Therefore the area of $\triangle ABC$ is $\frac{1}{2}(4x)(2y) = 4xy$.

$$\frac{\frac{1}{2}xy}{4xy} = \frac{1}{8}$$

□

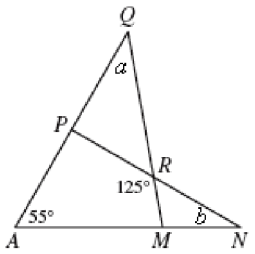
12. If $x + y = a$ and $x - y = b$, then what is the value of $2^{x^2-y^2}$?

(A) 2^{a+b}
(B) $2^{a^2-b^2}$
(C) 2^{a-b}
(D) $2^{a/b}$
(E) 2^{ab}

SOLN Factor $x^2 - y^2$ into $(x+y)(x-y)$; then $x^2 - y^2 = ab$. □

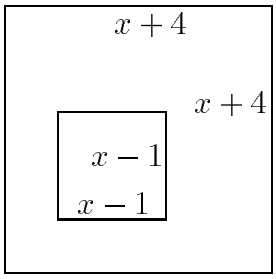
13. In the diagram, all lines that look straight are. What is the value of $a + b$?

(A) 55°
(B) 70°
(C) 75°
(D) 80°
(E) 90°



SOLN $m(\angle MRN) = 180° - 125° = 55°$ and $m(\angle PRQ) = 180° - 125° = 55°$. By the Exterior Angle Theorem, $m(\angle APR) = a + 55°$ and $m(\angle AMR) = b + 55°$. The sum of the interior angles of any quadrilateral $= 360°$, so $m(\angle PAM) + m(\angle APR) + m(\angle PRM) + m(\angle AMR) = 360°$. $\therefore$ $55° + (a + 55°) + 125° + (b + 55°) = 360°$. Solving for $a + b$ gives 70°. □

14. What is the area between the two squares if $x \geq 1$?



(A) $10x+15$

(B) $2x^2-4x+16$

(C) 25

(D) 3

(E) x^2+3x-4

SOLN The area between the squares is the area of the large square minus the area of the small square.

$$A=(x+4)^2-(x-1)^2=$$

$$x^2+8x+16-x^2+2x-1=$$

$$10x+15$$

□

15. $\square ABCD$ is a square with side length 2, M is the midpoint of $\overline{AB}$, and $\overline{MC}=\overline{ME}$. What is the length of $\overline{AE}$?

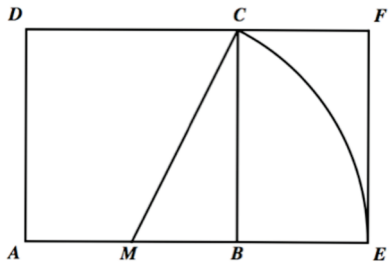
(A) $1+\sqrt{2}$

(B) $1+\sqrt{3}$

(C) $1+\sqrt{4}$

(D) $1+\sqrt{5}$

(E) $1+\sqrt{6}$



SOLN $\triangle BCM$ is a right triangle so the Pythagorean thrm gives $\overline{MC}=\sqrt{5}$.
$\overline{ME}=\overline{MC}=\sqrt{5}$.
Since $\overline{AB}=2$, then $\overline{AM}=1$.
Then $\overline{AE}=\overline{AM}+\overline{ME}=1+\sqrt{5}$. □

16.  is rotated to 

as  is rotated to

(A) 

(B) 

(C) 

(D) 

(E) 

SOLN The figure is rotated 90° cw looking down the positive z-axis. □

17. In working 4 hours, Frank made 24 machine parts. At this rate, how many parts will Frank make in 9 hours?

(A) 36 parts

(B) 45 parts

(C) 54 parts

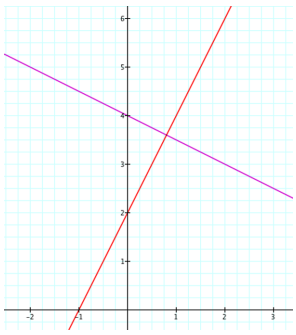
(D) 72 parts

(E) 81 parts

SOLN The rate is 24 parts in four hours = 6 parts per hour. □

18. What is the equation of the line perpendicular to $y = -\frac{1}{2}x + 4$ and passes through through the point $(2, 6)$?

(A) $y = 2x + 10$
(B) $y = \frac{1}{2}x + 5$
(C) $y = x + 4$
(D) $y = 2x + 2$
(E) $y = \frac{1}{2}x + \frac{1}{4}$

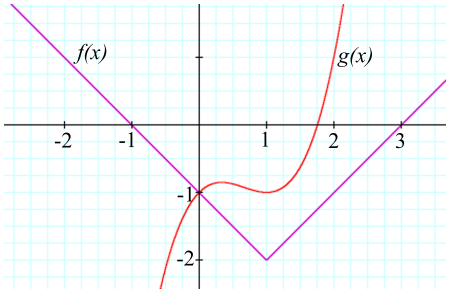


SOLN

The slopes of perpendicular lines are negative reciprocals of each other. □

19. What is $f(2) - g(1)$?

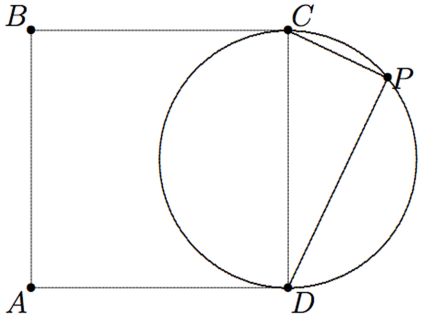
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2



SOLN $f(2) = -1$ and $g(1) = -1$ so $f(2) - g(1) = -1 - (-1) = 0$. □

20. In the diagram, $ABCD$ is a square and P is a point on the circle with diameter CD, $CP = 7$, and $PD = 11$. What is the area of the square?

(A) 170
(B) 220
(C) 240
(D) 310
(E) 335



SOLN Angle CPD is a right angle. $CD = \sqrt{7^2 + 11^2} = \sqrt{49 + 121} = \sqrt{170}$, and $CD^2 = 170$. □